

Unit 3
Populations and Samples

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1. A Feeling for Populations versus Samples

In unit 1, we restricted our attention to the data at hand. There was no consideration of the source of the data. We didn't ask questions such as "*what population did these data come from?*", "*how was the data obtained?*", "*was sampling used?*"?

- The **course introduction** included some examples making the point that statistics is a tool for evaluating uncertainty and that we must be wary of biases that might enter our thinking;
- In unit 1, *Summarizing Data*, the techniques and advantages of graphical and tabular summaries of data were emphasized; and
- In unit 1, *Summarizing Data*, too, we familiarized ourselves with the basics of summarizing data using numerical approaches (means, variances, etc)

If we broaden our attention, statistics is about

- To learn about and describe the population which gave rise to the sample; or
- To investigate some research question; or
- To assigning probabilities of events (mathematical tool); or
- To calculate the likelihood of the observed events under some specified postulates (statistical significance); and
- *What statistics is not about - Statistical significance is not the same as biological significance.*

The goal of this unit, *Populations and Samples*, is to develop an appreciation of the connections between populations and samples.

Example – The 1948 Gallup Poll

- Before the 1948 presidential election, Gallup polled 50,000
- Each was asked who they were going to vote for – Dewey or Truman

	Predicted by Gallup Poll	True Election Result
Dewey	50%	45%
Truman	44%	50%

- How could the prediction have been so wrong? Especially when $n=50,000$
- How could the sample have been so dissimilar to the population?

The 1948 Gallup Poll

- The actual sample turned out not to be representative of the population of actual voters
- It is thought that the error was the result of two things:

FIRST: The interviewers over-sampled

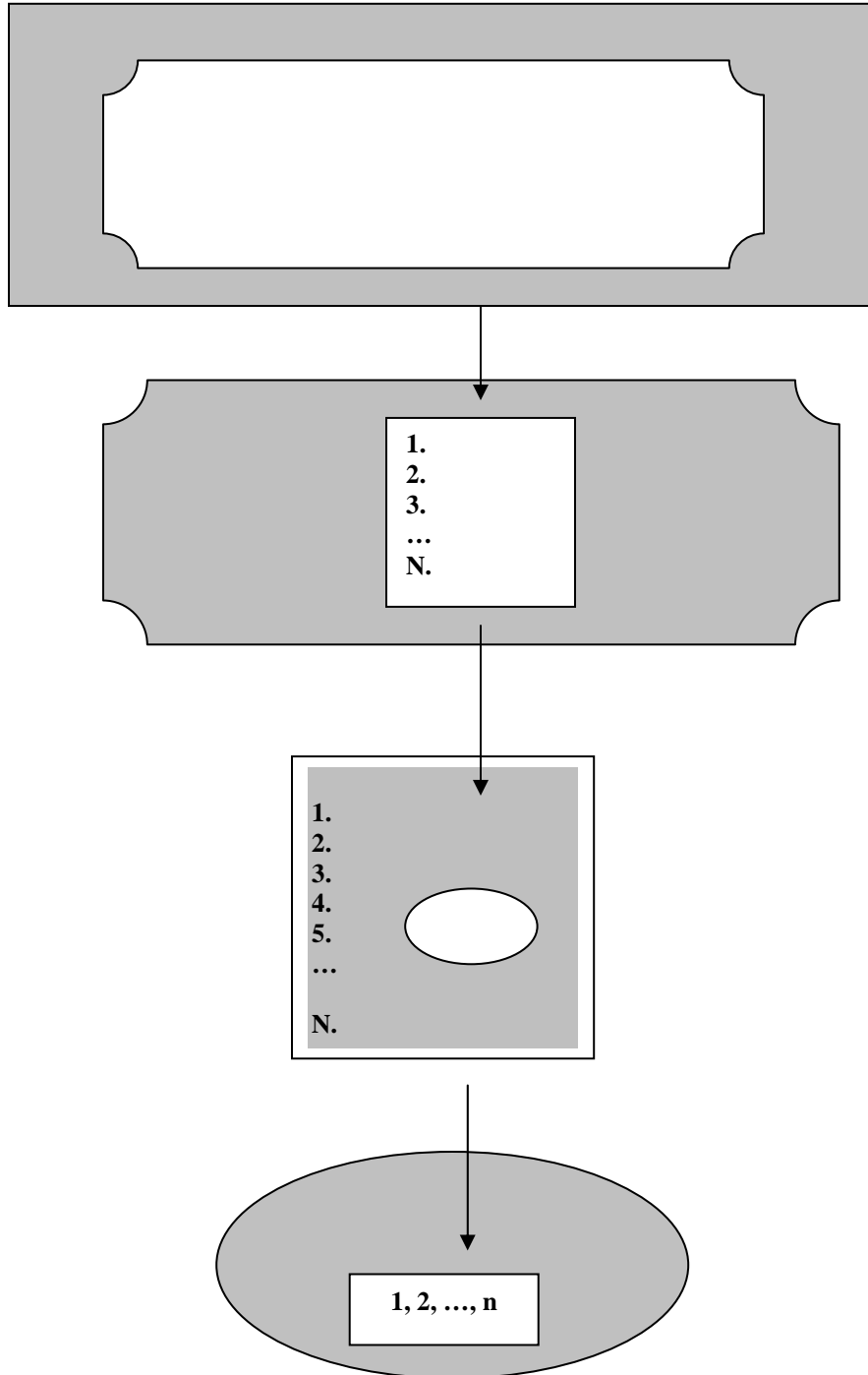
1. wealthy
2. safe neighborhoods, those with telephones AND . . .

SECOND: The over-sampled included a disproportionate number those favoring Dewey; ie - there was an over-sampling of the segment of the population more likely to vote for Dewey

Bias occurred not only because of over-sampling itself but also because the nature of the over-sampling was related to voter preference. *Note – Oversampling, per se, does not produce bias in study findings necessarily.*

*The population actually sampled (the sampled population)
Was not the same as the population of interest (the target population)*

2. Target Population, Sampled Population, Sampling Frame



Target Population
The whole group of interest.

Note – A convention is to use capital “N” to represent the size of a finite population.

Sampled Population
The subset of the target population that has at least some chance of being sampled.

Sampling Frame
An enumeration (roster) of the sampled population.

Sample
The individuals who were actually measured and comprise the available data.

Note – A convention is to use small “n” to represent the size of a sample.

<p>Target Population</p>	<ul style="list-style-type: none"> ● The aggregate of individuals who are of interest. ● <u>Example</u> – The population of 1948 presidential election voters who actually voted.
<p>Sampled Population</p>	<ul style="list-style-type: none"> ● The aggregate of individuals that was actually sampled. ● A listing of the entire sampled population comprises the <u>sampling frame</u>. ● GOAL: sampled = target ● The sampled population is often difficult to identify. We need to ask: Who did we miss? ● Constructing the sampling frame can be difficult ● <u>Example</u> – The 1948 Gallup poll sample is believed to have been drawn from the subset of the target population who were <ul style="list-style-type: none"> ♣ Easy to contact ♣ Consenting ♣ Living in safe neighborhoods

Sampling Frames Why They Are Difficult

To Construct a Sampling Frame Requires

- ♣ **Enumeration of every individual in the sampled population**
- ♣ **Attaching an identifier to each individual**
- ♣ **(Often, this identifier is simply the individual's position on the list)**

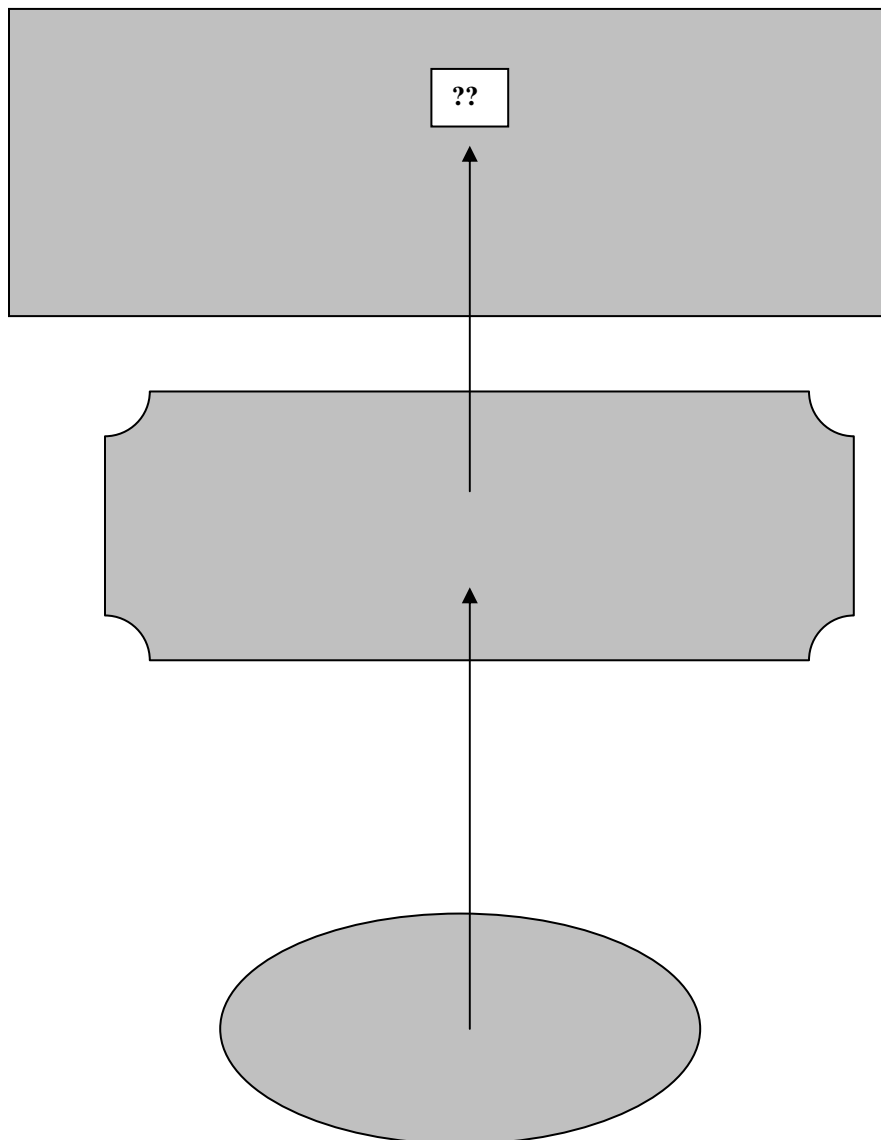
Example –

- ♣ **The League of Women's Voters Registration List might be the sampling frame for the target population who vote in the 2008 election.**
- ♣ **Individual identification might be the position on this list.**

Now You Try –

- ♣ **The target population is joggers aged 40-65 years.**
- ♣ **How might you define a sampling frame?**

3. On Making Inferences From a Sample (*this time - read from the bottom up*)



Target Population
The conclusion may or may not generalize to the target population.

- Refusals
- Selection biases

Sampled Population
If sampling is representative, then the conclusion applies to the sampled population

Sample
The conclusion is drawn from the sample.

- ♣ The conclusion is initially drawn from the sample.
- ♣ The question is then: **How far back does the generalization go?**
- ♣ The conclusion usually applies to the sampled population
- ♣ It may or may not apply to the target population
- ♣ The problem is: It is not always easy to define the sampled population

Example – an NIH Funded Randomized Trial

- ◆ **The sampling frame, by definition, is allowed to contain only consenters**

- ◆ **Thus, refusers, by definition, are not in the sampling frame.**

- ◆ **Accordingly, the sampled population differs from the target population in at least one respect, ALWAYS.**

- ◆ **This suggests that in any study, the preliminary analyses should always include a comparison of the consenters versus the refusers.**

Now You Try ...

- ◆ **Suppose the target population is current smokers.**

- ◆ **How might you construct a sampling frame?**

- ◆ **What do you end up with for a sampled population?**

- ◆ **Comment on the nature of generalization, to the extent possible.**

4. Simple Random Sampling

We would like our sampling plan to:

- ◆ permit generalization to the target population “in the long run” (unbiased);
- ◆ permit inferences that are not in error by too much, “in the short run” (minimum variance);

The virtue of simple random sampling:

- ◆ IF we draw sample after sample after sample after sample
AND IF, for each sample, we compute a sample \bar{X} as our guess of μ ,
so as to compile a collection of sample estimates \bar{X} ,
- ◆ THEN “in the long run”...
the average of all the sample estimates (\bar{X} after \bar{X} after \bar{X} ...)
will be equal to the population parameter value (the true value of μ)

“In the long run? What about the here and now?”

An individual sample will likely produce an estimate value that is NOT equal to the population value. It is for this reason that we seek out summary statistics that aren't too noisy (more on this in Unit 6 – Estimation).

Example of Simple Random Sampling
“Simple Random Sampling Without Replacement”

Suppose the Sampling Frame == Target Population

Subject ID	1	2	3	4	5	6
Age, years	21	22	24	26	27	36

The following is true for the population of size N=6

- ◆ Population mean age $\mu = \frac{21 + 22 + 24 + 26 + 27 + 36}{6} = 26$ years
- ◆ The investigator doesn't know this value. That's why s/he is taking a sample!

Sampling Protocol

- ◆ Include in study sample n=3 subjects drawn from the population at random and without replacement.

Calculation for Each Sample

- ◆ Sample mean $\bar{X} = \frac{\text{1st value} + \text{2nd value} + \text{3rd value}}{n = 3}$
- ◆ In this illustration (but not in real life) we can calculate the error of each \bar{X} by computing

$$\text{error} = 26 - \bar{X}$$

Consider all possible samples of size $n=3$ from a population of size $N=6$. There are 20 such samples.

Sample, $n=3$	Sample \bar{X}	Error = $26 - \bar{X}$
{ 21, 22, 24 }	22.333	+3.667
{ 21, 22, 26 }	23	+3
{ 21, 22, 27 }	23.333	+2.667
{ 21, 22, 36 }	26.333	-0.333
{ 21, 24, 26 }	23.667	+2.333
{ 21, 24, 27 }	24	+2
{ 21, 24, 36 }	27	-1
{ 21, 26, 27 }	24.667	+1.333
{ 21, 26, 36 }	27.667	-1.667
{ 21, 27, 36 }	28	-2
{ 22, 24, 26 }	24	+2
{ 22, 24, 27 }	24.333	+1.667
{ 22, 24, 36 }	27.333	-1.333
{ 22, 26, 27 }	25	+1
{ 22, 26, 36 }	28	-2
{ 22, 27, 36 }	28.333	-2.333
{ 24, 26, 27 }	25.667	+0.333
{ 24, 26, 36 }	28.667	-2.667
{ 24, 27, 36 }	29	-3
{ 26, 27, 36 }	29.667	-3.667

$$\frac{\sum \text{sample } \bar{X}}{\text{all 20 possible samples}} = \mu = 26$$

$$\sum_{\text{sample}\#1}^{\text{sample}\#20} [\text{error}] = 0$$

5. Some Non-Probability Sampling Plans

- ◆ Quota
- ◆ Judgment
- ◆ Volunteer

Quota Sampling Plan

Example –

Population is 10% African American

Sample size of 100 must include 10 African Americans

How to Construct a Quota Sample

1. Determine relative frequencies of each characteristic (e.g. gender, race/ethnicity, etc) that is hypothesized to influence the outcome of interest.
2. Select a fixed number of subjects of each characteristic (e.g. males or African Americans) so that

Relative frequency of characteristic in **sample** (e.g. 10%)

MATCHES

Relative frequency of characteristic in **population** (e.g. 10%)

Judgment Sampling Plan

Decisions regarding inclusion or non-inclusion are left entirely to the investigator. Judgment sampling is sometimes used in conjunction with quota sampling.

Example –

“Interview 10 persons aged 20-29, 10 persons aged 30-39, etc”
Sample size of 100 must include 10 African Americans.

Example –

Market research at shopping centers

Volunteer Sampling Plan

Volunteers are recruited for inclusion in the study by word of mouth, sometimes with an incentive of some sort (eg. gift certificate at a local supermarket)

Example –

For a study of a new diet/exercise regime, volunteers are recruited through advertising at local clinics, health clubs, media, etc.

Problem?

Limitations of a Non-Probability Sampling Plan

They're serious!

- 1. We have no idea if the sampling plan produces unbiased estimators. It probably doesn't.**

- 2. Any particular sample, by using fixed selection, may be highly unrepresentative of the target population.**

- 3. Statistical inference, by definition based on some sort of probability model, is not possible.**

- 4. Regarding quota sampling**
 - We have no real knowledge of how subjects were selected (recall the Gallup poll example)

- 5. Regarding judgment sampling**
 - Likely, there is bias in the selection – at least for the reasons of comfort and convenience

- 6. Regarding volunteer sampling**
 - Volunteers are likely to be a “select” group for being motivated

6. Introduction to Probability Sampling Plans

The way to go.

A **probability sampling plan** is “out of the hands” of the investigator.

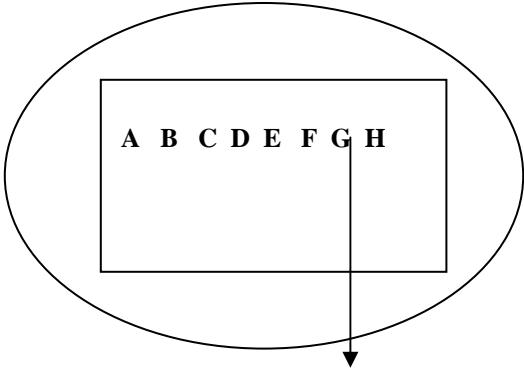
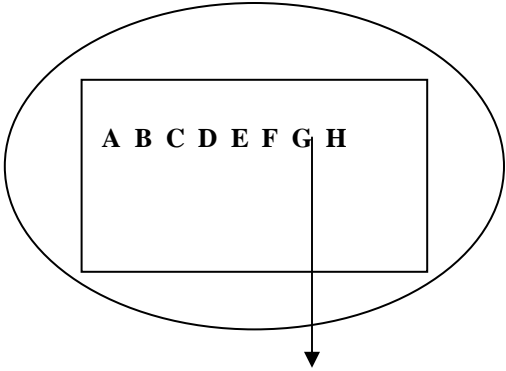
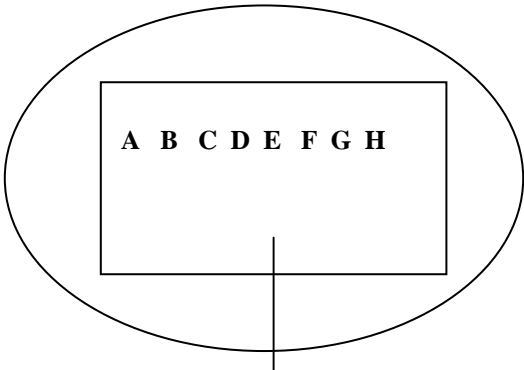
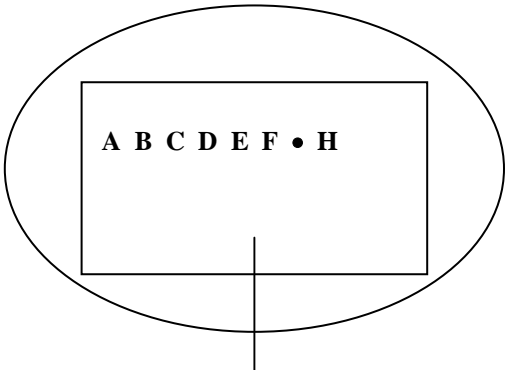
- ◆ Each individual has a known probability of inclusion in the sample, prior to sampling.
- ◆ The investigator has no discretion regarding the inclusion or exclusion of an individual
- ◆ This eliminates one source of potential bias – that on the part of the investigator.

How do we know if a sample is REPRESENTATIVE?

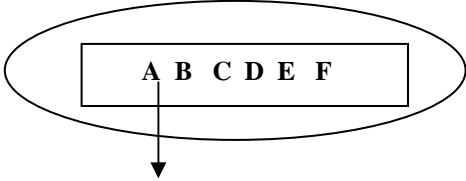
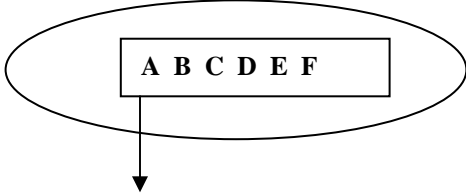
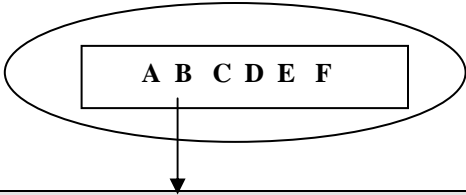
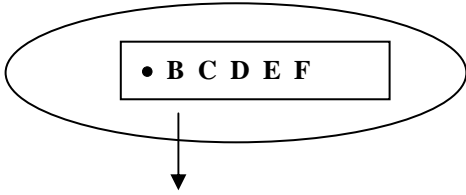
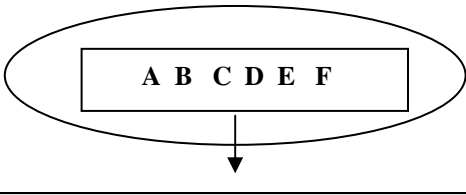
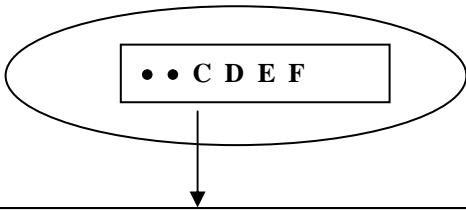
- ◆ Ultimately, we don't know.
- ◆ So, instead, we use an unbiased sampling plan and hope for the best.
- ◆ In the meantime, we can generate some descriptive statistics and compare these to what we know about the population.

Simple Random Sampling is the basic probability sampling method.

Sampling *With* Replacement
versus
Sampling *Without* Replacement

With Replacement	Without Replacement
	
<p>1st Draw: A B C D E F G H are available</p> <p>- Suppose “G” is selected for inclusion</p>	<p>1st Draw: A B C D E F G H are available</p> <p>- Suppose “G” is selected for inclusion</p>
	
<p>2nd Draw: A B C D E F G H are ALL available</p> <p>- Thus “G” is available for inclusion a 2nd time.</p>	<p>2nd Draw: A B C D E F H are available but “G” is not available anymore</p> <p>- Thus, “G” can only be included once.</p>
<p>Etc</p>	<p>etc</p>

Example –
What is the Probability of {A B C}?

With Replacement	Without Replacement
	
Probability {A} = 1/6	Probability {A} = 1/6
	
Probability {B given A} = 1/6	Probability {B given A} = 1/5
	
Probability {C given A, B} = 1/6	Probability {C given A,B} = 1/4
Probability{sample=A,B,C}=(1/6)(1/6)(1/6)	Probability{sample=A,B,C}=(1/6)(1/5)(1/4)

Sampling With versus Without Replacement
General

With Replacement	Without Replacement
<p>Population size = N Sample size = n</p>	<p>Population size = N Sample size = n</p>
<p>Each individual has a 1/N chance of inclusion in the sample.</p>	<p>Each individual has a 1/N chance of inclusion in the sample, overall.</p>
<p>Total # possible samples of size = n is</p> <p>(N) (N) (N) ... (N) = Nⁿ</p> <p>↑ ↑ ↑ ↑</p> <p>1st 2nd 3rd nth</p> <p>draw draw draw draw</p>	<p>Total # possible samples of size = n is</p> <p>(N) (N-1) (N-2) (N-n+1)</p> <p>↑ ↑ ↑ ↑</p> <p>1st 2nd 3rd nth</p> <p>draw draw draw draw</p>
<p>Probability {each sample of size n} =</p> $\frac{1}{N^n}$	<p>Probability {each sample of size n} =</p> $\frac{1}{N(N - 1)(N - 2) \dots (N - n + 1)}$

Example
Sampling WITH Replacement

Population

Four Queens in a deck of cards

Sampling Plan

- Draw one card at random
- Note its suit
- *Return the selected card*
- Draw one card at random
- Note its suit

Population size, $N=4$

Sample size, $n=2$

Total # samples possible = $(4)(4) = 4^2 = N^n = 16$

Probability of each sample = $\frac{1}{N^n} = \frac{1}{16}$

Here are the 16 possible samples:

(spade, spade)	(spade, club)	(spade, heart)	(spade, diamond)
(club, spade)	(club, club)	(club, heart)	(club, diamond)
(heart, spade)	(heart, club)	(heart, heart)	(heart, diamond)
(diamond, spade)	(diamond, club)	(diamond, heart)	(diamond, diamond)

What if the Order of the Sample Doesn't Matter?
*This applies to the **Binomial Distribution** to Come...*

Example –

{ A B C } is the same as
 { A C B } is the same as
 { B A C } is the same as
 { B C A } is the same as
 { C A B } is the same as
 { C B A }

How many ways are there to order “A”, “B”, and “C”

- # letter choices for the first position = 3
- Having specified a selection for first position,
letter choices for the second position = $(3 - 1) = 2$.
- Having specified selections for the first and second positions,
letter choices for the third position = $(3 - 1 - 1) = 1$.
- Therefore the # ways to order “A”, “B”, “C” = $(3)(3-1)(3-2) = 6$

How many ways are there to order “A”, “B”, …, “n”?

- # letter choices for the first position = n
- Having specified a selection for first position,
letter choices for the second position = $(n - 1)$.
- Having specified selections for the first and second positions,
letter choices for the third position = $(n - 2)$, etc
- Therefore the # ways to order “A”, … “n” = $(n)(n-1)(n-2) \cdots (2)(1)$

*This is called the number of **permutations** of 1, 2, …, n*

Example
Sampling WITHOUT Replacement
Order Does NOT Matter

Population

Four Queens in a deck of cards

Sampling Plan

- Draw one card at random
- Note its suit
- *Set the selected card aside (Do NOT return it to the pile)*
- Draw another card at random
- Note its suit

Population size, $N=4$

Sample size, $n=2$

Total # samples possible = $(4)(4-1) = (4)(3) = 12$

Probability of each sample, ordered = $\frac{1}{N(N-1)} = \frac{1}{(4)(3)} = \frac{1}{12}$

Here are the 12 possible ordered samples:

(spade, club)	(heart, club)	(diamond, heart)
(club, spade)	(club, heart)	(heart, diamond)
(heart, spade)	(diamond, club)	(spade, diamond)
(spade, heart)	(club, diamond)	(diamond, spade)

Notice that each “net” result of two cards has two possible orderings.

**By inspection, we can see easily that
Probability {1 club and 1 spade, regardless of order} = 2/12**

More generally –

$$\left(\begin{array}{l} \text{Total \#} \\ \text{ordered samples} \end{array} \right) = \left(\begin{array}{l} \text{\# ways to} \\ \text{draw 1st} \end{array} \right) \left(\begin{array}{l} \text{\# ways to} \\ \text{draw 2nd} \end{array} \right)$$

$$= (N)(N - 1)$$

$$= (4)(3)$$

$$= 12$$

$$\left(\begin{array}{l} \text{\# orderings of} \\ \text{given sample} \end{array} \right) = \left(\begin{array}{l} \text{\# choices for} \\ \text{position 1} \end{array} \right) \left(\begin{array}{l} \text{\# choices for} \\ \text{position 2} \end{array} \right)$$

$$= (n)(n - 1)$$

$$= (2)(1)$$

$$= 2$$

$$\left(\begin{array}{l} \text{Probability of} \\ \text{1 club and 1 heart} \end{array} \right) = \frac{\text{\# orderings of a "net" result}}{\text{total \# of ordered samples}} = \frac{(n)(n - 1)}{(N)(N - 1)} = \frac{(2)(1)}{(4)(3)} = \frac{2}{12}$$

Sampling **WITHOUT** Replacement Summary

IF Order **DOES** Matter

- ◆ **Total # ordered samples = $N(N-1)(N-2) \cdots (N-n+1)$**
- ◆ **Probability of each ordered sample = $\frac{1}{(N)(N-1)\dots(N-n+1)}$**

IF Order Does **NOT** Matter

- ◆ **Total # ordered samples = $N(N-1)(N-2) \cdots (N-n+1)$**
- ◆ **# ways to order (“permute”) each “net” result of size $n = (n)(n-1)(n-2)\dots(2)(1)$**
- ◆ **Total # of Unordered samples =**

$$\frac{\text{Total \# ordered samples}}{\text{\# orderings of 'net' result}} = \frac{(N)(N-1)(N-2)\dots(N-n+1)}{(n)(n-1)\dots(2)(1)}$$

- ◆ **Probability of each “net” unordered sample = $1/(\text{total \# unordered samples}) =$**

$$\frac{\text{\#orderings of the "net" result}}{\text{Total \# ordered samples}} = \frac{(n)(n-1)(n-2)\dots(2)(1)}{(N)(N-1)\dots(N-n+1)}$$

How to Select a Simple Random Sample **WITHOUT Replacement**
(Using a random number table)

<p><u>Step 1:</u></p> <p>List the subjects in the sampled population.</p> <ul style="list-style-type: none"> ♣ This is the <u>sampling frame</u>. 	<p><u>Example</u> -</p> <p>Select a simple random sample of 30 subjects from a population of 500</p>
<p><u>Step 2:</u></p> <p>Number this listing from “1” to “N”</p> <ul style="list-style-type: none"> ♣ where N = size of sampled population 	<p><u>Example, continued</u> -</p> <p style="text-align: center;">N = 500 n = 30</p>
<p><u>Step 3:</u></p> <p>The size of “N” tells you how many digits in a random number to be looking at:</p> <ul style="list-style-type: none"> ♣ For $N \leq 10$ Need only read 1 digit ♣ For N between 10 and 99 Read 2 digits ♣ For N between 100 and 999 Read 3 digits <p style="text-align: center;">etc</p>	<p><u>Example, continued</u> –</p> <p>N=500 is between 100 and 999</p> <ul style="list-style-type: none"> ♣ For N between 100 and 999 Read 3 digits

Step 4:

Using the random number table,
pick a random number as a starting point

79889	75532	28704
48895	11196	34335
89604	41372	10837

**Example, continued -**

The first 3 digits of this number is “111”.
So we will include the 111th subject in our sample

Step 5:

Proceed down your selected column of the random number table, row by row.
With each row, if the required digits are $\leq N$, INCLUDE
With each row, if the required digits are $> N$, PASS BY
With each row, if the required digits are a repeat of a previous selection, PASS BY

79889	75532	28704
48895	11196	34335
89604	41372	10837

**Example, continued -**

The first 3 digits of the second random number is “413”.
So we will include the 413th subject in our sample

Step 6:

Repeat “Step 5” until you have included the required number ($n=30$) in your sample

Remarks on Simple Random Sampling

Advantages:

- Selection is entirely left to chance.
- Selection bias is still possible, but chances are small.
- No chance for discretion on the part of the investigator or on the part of the interviewers.
- We can compute the probability of observing any one sample. This gives a basis for statistical inference to the population, our ultimate goal.

Disadvantages:

- We still don't know if a particular sample is representative
- Depending upon the nature of the population being studied, it may be difficult or time-consuming to select a simple random sample.
- An individual sample might have a disproportionate # of skewed values.

7. Some Other Probability Sampling Plans

- ◆ **Systematic**
- ◆ **Stratified**
- ◆ **Multi-stage**

Systematic Sampling

- **Population size = N**
- **Desired is an $(n/N) = X$, or a $100X\%$ sample**
- **Pick the first item by simple random sampling.**
- **Thereafter, select every $(1/X)$ th item**
Example: $n=20, N=100 \rightarrow (n/N) = 20/100 = .05$
or $X=.05$, for a $100(.05)=5\%$ sample \rightarrow select $1/.05=20$, or every 20th item
- **Probability an item is included on the 1ST draw = $1/N = .01$**
- **Given the 1ST item has been drawn, probability an item is included on any other draw is 0 or 1.**

Example –

Suppose we want a sample of size $n=100$ from the $N=1000$ medical charts in a clinic office.

Pick the 1ST chart by simple random sampling.

$$n/N = 100/1000 = .10 \rightarrow 1/.10 = 10$$

Thereafter, select every 10th chart.

Remarks on Systematic Sampling

Advantages:

- It's easy.
- Depending on the listing, the sampled items are more evenly distributed.
- As long as there is no association with the order of the listing and the characteristic under study, this should yield a representative sample.

Disadvantage:

- If the sampling frame has periodicities (a regular pattern) and the rule for systematic sampling happens to coincide, the resulting sample may not be representative.

Example of a Periodicity that Results in a Biased Sample:

- Clinic scheduling sets up 15 minute appointments with physicians
- Leaves time for an emergency, or walk-in visit at 15 minutes before the hour, every hour.
- Doing a chart-audit, you sample every 4th visit and get only the emergency visits selected into the sample, or else none of them.

Stratified Sampling
Simple Random Sampling within Strata

Example-

Do construction workers experience major health problems?

Do health problems differ among males and females?

Construction workers, as a group, are likely to be comprised predominately of males.

Thus, if we take a simple random sample we may get very few women in the sample.

Procedure:

1. Define mutually exclusive strata such that the outcome of interest is likely to be

**similar within a stratum; and
very different between strata.**

outcome: health problems
strata: Males / Females

2. Obtain a simple random sample from each stratum

**We want to be sure to get a good overall sample.
Sampling each stratum separately ensures this.**

Remarks on Stratified Sampling

Advantage:

- Good when population has high variability, especially when the population includes a mix of people (eg. males and females) that are NOT similarly represented (eg. population is disproportionately male)

Take care:

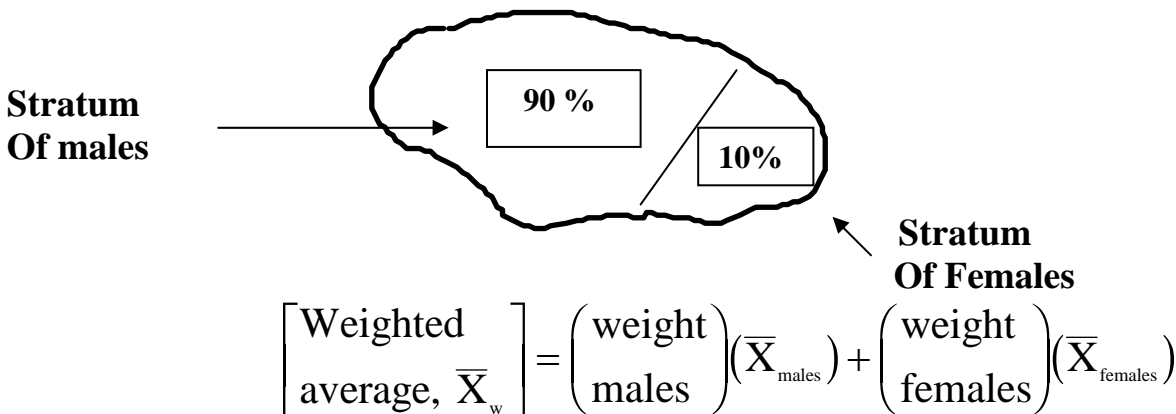
- Strata MUST be mutually exclusive and exhaustive
- To compute an overall population estimate requires use of weights that correspond to representation in the population. Following is an example.

Example of Calculation of Weighted Mean from a Stratified Sample -

Goal – To estimate the average # cigarettes smoked per day among all construction workers.

Population is disproportionately male (90% male, 10% female)

- ♣ Weight given to average observed for males = 0.90
- ♣ Weight given to average observed for females = 0.10
- ♣ Note that weights total 1.00



Multi-Stage Sampling
Good, Sometimes essential, for Difficult Populations

Example Suppose we want to study the gypsy moth infestation.

A **multistage** sample plan calls for

- 1ST - Select individual trees
(Primary sampling units - PSU's)

- 2nd - Select leaves from only the selected trees
(Secondary sampling units)

Multistage Sampling

- The selection of the primary units may be by simple random sampling

- The selection of the secondary units may also be by simple random sampling

- Inference then applies to the entire population

CAUTION!!!

- Take care that the selection of primary sampling units is NOT on the basis of study outcome. Bias would result.

7. The Nationwide Inpatient Survey (NIS) *Sampling Designs Can Be Quite Complex*

