## Topic 4 - Bernoulli and Binomial <br> Self Evaluation Quiz <br> SOLUTIONS

1. According to a recent poll, $41 \%$ of US citizens approve of the job President Bush is doing. Assume that this proportion is actually true for the whole of the US population.
(a) Suppose a simple random sample of 10 citizens is selected. What is the probability that a majority (more than 5) approve of the job that President Bush is doing?

Answer: . 18

## Solution:

Define X is the number of approve. We need to find $\mathrm{P}(\mathrm{X}>5)$.
Recall that binomial distribution is given by
$\mathrm{P}(\mathrm{X}=\mathrm{x})=\binom{n}{x} p^{x}(1-p)^{n-x}$, where p is the population parameter for the probability of success, $n$ is the number of trials and $x$ is the number of successes.

We have, $\mathrm{p}=0.41, \mathrm{n}=10$ and $\mathrm{x}>5$.

$$
\begin{aligned}
& \mathrm{P}(\mathrm{X}>5)=\mathrm{P}(\mathrm{x}=6)+\mathrm{P}(\mathrm{X}=7)+\mathrm{P}(\mathrm{x}=8)+\mathrm{P}(\mathrm{X}=9)+\mathrm{P}(\mathrm{x}=10) \\
& =\binom{10}{6}(0.41)^{6}(1-0.41)^{10-6}+\binom{10}{7}(0.41)^{7}(1-0.41)^{10-7}+\binom{10}{8}(0.41)^{8}(1-0.41)^{10-8}+ \\
& +\binom{10}{9}(0.41)^{9}(1-0.41)^{10-9}+\binom{10}{10}(0.41)^{10}(1-0.41)^{10-10} \\
& =0.1834
\end{aligned}
$$

(b) Suppose a simple random sample of 50 citizens is selected. What is the probability that a majority (more than 25) approve of the job that President Bush is doing?

Answer: . 08
Solution:
$\mathrm{P}(\mathrm{X}>25)=0.0761$
This result can be obtained as in part (a). A more practical approach would be to use a probability calculator applet on http://www.stat.tamu.edu/~west/applets/binomialdemo.html

Here is a snapshot of the result using the above website:


How it works: The calculator above takes the place of the traditional textbook table. Students should enter the proper binomial parameters ( $n$ and $p$ ) for the distribution they are interested in calculating probabilities for. Students specify the relevant "x" value and then select among choices such as "exactly", "no more than", etc. Hit "Compute!" to get the answer. The probability is represented graphically in the plot.

Also notice that the peak corresponds to the mean $=n p=50 * 0.41 \approx 21$.
2. The probability that a person suffering from a migraine headache will obtain relief from a particular drug is 0.9. Three randomly selected sufferers from migraine headache are given the drug. Find the probability that the number obtaining relief will be 2 or 3 .

Answer: . 97

## Solution:

$\mathrm{P}(\mathrm{X}>=2)=\mathrm{P}(\mathrm{x}=2)+\mathrm{P}(\mathrm{x}=3)=0.9720$

$$
\mathrm{P}(\mathrm{X}>=2)=\binom{3}{2}(0.9)^{2}(1-0.9)^{3-2}+\binom{3}{3}(0.9)^{3}(1-0.9)^{3-3}=0.9720
$$

3. A home security system has a $80 \%$ reliability rate, meaning that it goes off $80 \%$ of the time when there is a burglary. Suppose that 12 homes equipped with this system experience an attempted burglary. What is the probability that more than 7 alarms go off?

Answer: . 93

## Solution:

$\mathrm{P}(\mathrm{X}>7)=0.9274$

$$
\mathrm{P}(\mathrm{X}>7)=\mathrm{P}(\mathrm{X}=8)+\mathrm{P}(\mathrm{X}=9)+\mathrm{P}(\mathrm{X}=10)+\mathrm{P}(\mathrm{X}=11)+\mathrm{P}(\mathrm{X}=12)
$$

Use the Java applet given here http://www.stat.tamu.edu/~west/applets/binomialdemo.html


How it works: The calculator above takes the place of the traditional textbook table. Students should enter the proper binomial parameters ( $n$ and $p$ ) for the distribution they are interested in calculating probabilities for. Students specify the relevant "x" value and then select among choices such as "exactly", "no more than", etc. Hit "Compute!" to get the answer. The probability is represented graphically in the plot.
4. (Source: Virtual Lab) . The common form of hemophilia is due to a defect on the X chromosome (one of the two chromosomes that determine gender). We will let " $h$ " denote the defective gene, linked to hemophilia, and H the corresponding normal gene. Women have two X chromosomes, and " $h$ " is recessive. Thus, a woman with gene type HH is normal, a woman with gene type "hH" or "Hh" is free of disease but is a carrier; and a woman with gene type "hh" has the disease. A man has only one X chromosome (the other sex chromosome, the Y chromosome, plays no role in the disease. A man with gene type $h$ has hemophilia and a man with gene type H is healthy.

| Mother XX | Father X |
| :--- | :--- |
| HH - Normal | H - Normal |
| Hh - Carrier | h - Hemophilia |
| hH - Carrier |  |
| hh - Hemophilia |  |

(a) Suppose that a mother is a carrier and the father is healthy. They have a son. What is the probability that the son will have hemophilia? Will be healthy?

Answer: $\operatorname{Pr}[$ son is hemophiliac] = $5 \operatorname{Pr}[$ son is healthy $]=.5$
Solution:
If child is a son, then father contributed a Y chromosome. The Y plays no role in disease. Thus son's chances of H and h are both 0.5
(b) Suppose that a mother is a carrier and the father has hemophilia. They have a daughter. What is the probability that the daughter will have hemophilia? Will be a carrier?

Answer: $\operatorname{Pr}[$ daughter is hemophiliac $]=.5 \operatorname{Pr}[$ daughter is carrier] = 5
Solution:
If child is a daughter, then father contributed an X chromosome. If it is known that the father is a hemophiliac, then we know he contributed an $h$ with probability 1 . If the mother is a carrier, then she is hH (or other way around) plays no role in disease. Thus daughter is hh or hH , each with probability .5

