



Supporting Online Material for  
**Measurement of Forces Inside a Three-Dimensional Pile of Frictionless Droplets**

J. Zhou, S. Long, Q. Wang, A. D. Dinsmore\*

\*To whom correspondence should be addressed. E-mail: [dinsmore@physics.umass.edu](mailto:dinsmore@physics.umass.edu)

Published 16 June 2006, *Science* **312**, 1631 (2006)  
DOI: 10.1126/science.1125151

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Materials and Methods  
Fig. S1

# Measurement of Forces inside a Three-Dimensional Pile of Frictionless Droplets,

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## *The statistical model:*

To implement our model, we consider a single droplet with  $Z$  contact forces, each acting in a direction that passes through the droplet's center of mass (*i.e.*, there are only normal forces and no torque). The directions of the  $Z$  contacts are randomly chosen, except for the constraint that the angles between any two contacts,  $\theta$ , must exceed a minimum value,  $\theta_{\min}$  (as seen in the data). This minimum angle constraint arises from the fact that two neighboring contacts are separated by the sizes of the corresponding droplets;  $\theta_{\min} = 60^\circ$  for monodisperse piles but is smaller for polydisperse ones. Starting with an arbitrary initial probability distribution,  $P$ , of contact forces on one droplet, we randomly draw the first ( $Z-3$ ) of forces from  $P$ . The remaining three forces are computed by setting the three components of the force to zero. A complete configuration of all  $Z$  forces is thus obtained. We collect only those force configurations that contain only non-negative forces because of the absence of adhesion between droplets in the experiment. From this subset of configurations, we calculate a new force probability distribution,  $P$ . Repeating the above process with the new  $P$  as input, we achieve a converged probability distribution  $P$  after several iterations. The final result does not depend on the functional form of the initial  $P$ . The distributions of angles  $\theta$  and  $\theta_f$  were not strongly dependent on  $Z$ . The results for  $Z = 6$  are shown in Fig. 3 (A, C) of the text. We emphasize that this model assumes isotropic loading.

## *Strain hardening:*

We measured the compressive strain of emulsions of different heights subjected to stress applied to the top by means of Teflon weights immersed in the fluid and resting on the droplets. The compressive strain  $\varepsilon$  is defined as the change in the pile's height normalized by the height without added weights. Linear response was found for small  $\varepsilon$  and nonlinear response (strain hardening) occurred at larger  $\varepsilon$  (Fig. S1). In a 5-mm-tall pile, strain hardening was observed for  $\varepsilon >$  approximately 0.15. In a 300- $\mu\text{m}$ -tall pile, strain hardening was more pronounced and occurred at a smaller strain of approximately 0.08.

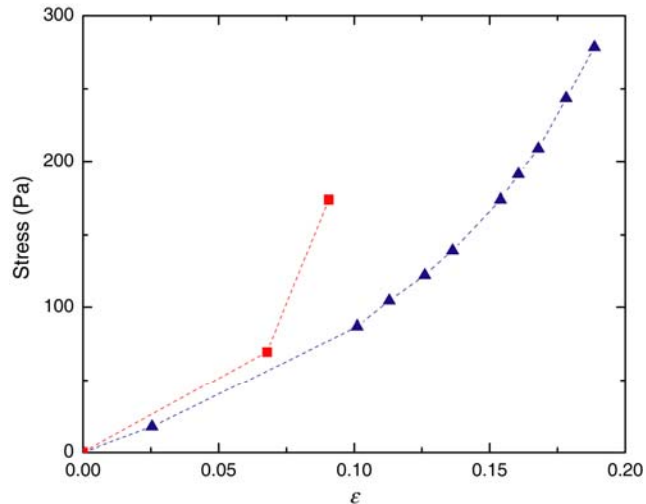


Fig. S1. Stress vs. strain curves for a 5-mm-tall polydisperse emulsion pile (-- $\blacktriangle$ --) and a short polydisperse emulsion pile of 300  $\mu\text{m}$  (-- $\blacklozenge$ --).