BIO744
HOMEWORK 2
DUE IN CLASS: TUESDAY, OCTOBER 9

Christensen: Exercises B.11, B.17, 1.1, 1.2, 1.3

1. Prove that for any matrix X, r(X'X)=r(X)
2. Consider the linear model

$$Y = X\beta + \epsilon$$

where $E(\epsilon) = 0$, $Cov(\epsilon) = \sigma^2 I$, $Y = (y_1, y_2, y_3)'$, $\beta = (\beta_1, \beta_2, \beta_3)'$ and

$$X = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

(a) Compute $r(X)$
(b) Find $C(X)$, $C(X)^{\perp}$ and their dimensions
(c) Construct an orthonormal basis for $C(X)$
(d) Compute the unique orthogonal projection operator onto $C(X)$