# A New Account of Quantifiers

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# 1. Introduction

We have a theory of pronoun-binding, but like all extant theories of pronoun-binding, it faces numerous difficulties – both methodological and empirical. We also have a theory of logical-conjunction (&), which is *mostly* adequate, but nevertheless faces its own difficulties.

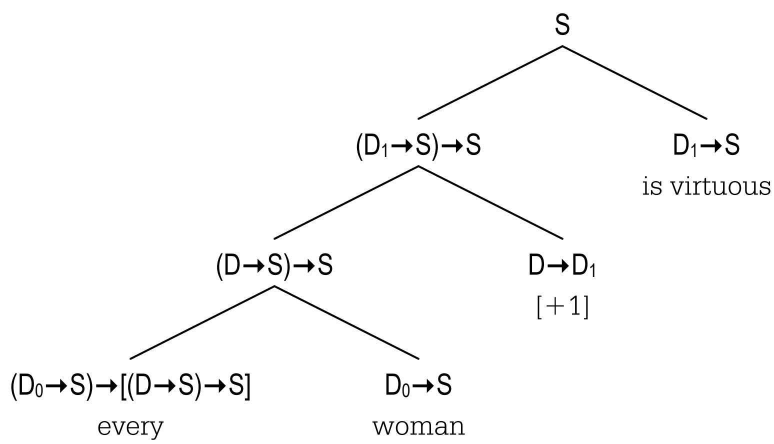
As it turns out, both of these problems can be solved by a new theory of quantifiers.[1]

# 2. Our Old Theory of Quantifiers

According to our current theory, which traces to Montague, a quantifier is a two-place second-order predicate, which in particular has the following type.[2]

$$(D_0 \rightarrow S) \rightarrow [(D \rightarrow S) \rightarrow S]$$

In other words, a quantifier takes a common-noun phrase (0-inflected predicate), and delivers a function that takes an uninflected predicate, and delivers a sentence. The following syntactic-tree illustrates.




The intermediate phrase 'every woman' is what we call a ***quantifier-phrase***, which is a species of NP, which is an über-category. Basically, an NP is the sort of phrase that can be case-marked, and can be cross-referenced by a pronoun. For example, in the above tree 'every woman' is marked nominative [+1]. On the other hand, the phrase 'is virtuous' – whose further syntactic structure is ignored above – is a VP, which is a one-place predicate that sub-categorizes for a single nominative argument. Also note that, in keeping with the second-order nature of quantifiers and quantifier-phrases, the QP 'every woman' takes the VP 'is virtuous' as an argument.

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[1] The finitary versions of the quantifiers are connectives, which we examine in a later chapter.

[2] Here, D is the type of definite-noun-phrases (a.k.a. DNPs), and S is the type of (closed) sentences; alternatively, D is the type of entities (underlying elements of the domain), and S is the type of statements. We reserve the type E for events, which are employed to interpret eventive-verbs. [So far, our verbs are all stative.]

## 3. Our New Theory of Quantifiers

We propose an alternative account of quantifiers in terms of a new class of type-theoretic objects, which we call ***junctions***, which are infinitary-operators.[3] All told (or perhaps, all *tolled*), we will propose six different junctions, as follows.

| | | |
|---|---|---|
| ⋀ | conjunction | every |
| ⋁ | disjunction | some |
| Л | subjunction | any |
| Σ | sum | "a" (basic)[4] |
| Π | product | "a" (promoted) |
| ⊠ | exclusive-disjunction | exactly-one |

## 4. 'Every' – Conjunction

We start with the simplest junction, which is infinitary-conjunction (⋀). First, we posit a new type-forming operator ⋀ – called 'conjunction',[5] defined so that

if $\mathfrak{I}$ is a type, then so is $\bigwedge\mathfrak{I}$.

This means that the following are examples of types.

| | |
|---|---|
| $\bigwedge$D | conjunctions of D's |
| $\bigwedge$S | conjunctions of S's |
| $\bigwedge$(D→S) | conjunctions of predicates |
| $\bigwedge$(S→S) | conjunctions of connectives |
| $\bigwedge$D → S | functions from conjunctions of D's to S's |
| $\bigwedge$(D→S) → D | functions from conjunctions of predicates to D's |

As with type-theory in general, although there is a staggering array of types, only a tiny fraction are in fact instantiated by any particular natural language.

We further propose the following type-identity

$\bigwedge$S = S

which tracks the intuition that the conjunction of any number of sentences is itself a sentence.

There is a corresponding expansion of the syntax of type-theory, which now posits all expressions of the following form.

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[3] By an infinitary-operator, we mean an operator that acts on *a collection of arguments* of arbitrary size, even infinite, and even empty.

[4] The word 'a' is scare-quoted because it is often not pronounced. In English, singular common-noun-phrases *require* 'a', but plural-phrases and mass-phrases *prohibit* 'a'. By comparison, Spanish has plural forms of 'a' ['unas' and 'unos'].

[5] It is also called 'meet', which is borrowed from lattice theory, which borrows it (in effect) from projective geometry. Lattice-theoretic meet (a.k.a. *greatest lower bound*) is a generalization of logical-conjunction and set-intersection, and also "where two subspaces meet".

$\wedge_\nu\{\varepsilon \mid \Phi\}$

which we propose to read as:

the conjunction [over ν] of all ε such that Φ

Here, ν is any variable, ε is any expression of type $\mathfrak{I}$, Φ is any formula, and the resulting expression has type $\wedge\mathfrak{I}$. Note that the variable ν officially identifies which variable is bound. On the other hand, in most applications, exactly one variable is free in both ε and Φ, in which case it is clear which one is bound. In these circumstances, we omit ν, in which case we have (informal) expressions of the following form,

$\wedge\{\varepsilon \mid \Phi\}$

which we read:

the conjunction of all ε such that Φ.

## 5. Quantifiers and Quantifier-Phrases

Our new type-logical apparatus allows us to reformalize quantifiers and quantifier phrases. First, we read

$\wedge\{x \mid \Phi\}$

as

the conjunction of all $x$ such that Φ.

So, for example, we can read

$\wedge\{x \mid x \text{ is a man}\}$

as

the conjunction of all $x$ such that $x$ is a man

or more succinctly:

the conjunction of all men.

Intuitively, this amounts to

$\text{man}_1$ and $\text{man}_2$ and … and $\text{man}_k$

where $\{\text{man}_1, \ldots, \text{man}_k\}$ is the class of all men.

With this in mind, we offer our new proposal.

$$[\![\text{every}]\!] \quad = \quad \lambda P_0 \wedge\{x \mid Px\}$$

For example:

$$\begin{aligned} [\![\text{every man}]\!] \quad &= \quad \wedge\{x \mid x \text{ is a man}\} \\ &= \quad \text{the conjunction of all men} \\ &\approx \quad \text{man}_1 \text{ and } \text{man}_2 \text{ and } \ldots \text{ and } \text{man}_k \end{aligned}$$

# 6. Examples

1. every man is virtuous

<table>
<tr><td></td><td>every</td><td>man</td><td>[+1]</td><td>is</td><td>virtuous</td><td></td></tr>
<tr><td></td><td>$\lambda P_0 \wedge\{x \mid Px\}$</td><td>$\mathbf{M}_0$</td><td rowspan="2">$\lambda x\{x_1\}$</td><td>$\lambda P_0\{P_1\}$</td><td>$\mathbf{V}_0$</td><td></td></tr>
<tr><td>①</td><td colspan="2">$\wedge\{x \mid \mathbf{M}x\}$</td><td colspan="2" rowspan="2">$\mathbf{V}_1$</td><td rowspan="2">②</td></tr>
<tr><td>③</td><td colspan="3">$\wedge\{x_1 \mid \mathbf{M}x\}$</td></tr>
<tr><td>④</td><td colspan="5">$\wedge\{\mathbf{V}x \mid \mathbf{M}x\}$</td><td></td></tr>
<tr><td>⑤</td><td colspan="5">$\forall x \{ \mathbf{M}x \rightarrow \mathbf{V}x \}$</td><td></td></tr>
</table>

**Explanation of Computations:**

(1) Compositions ① and ② involve function-application, in accordance with *expanded* lambda-conversion. Nothing new here.

(2) Compositions ③ and ④ appeal to the following new composition principle.[6]

| **∧-composition** | |
|---|---|
| $\alpha$ | $\alpha$ is any expression, or null |
| $\wedge_\nu\{\beta \mid \Phi\}$ | |
| $\{\alpha,\beta\} \vdash \gamma$ | a sub-derivation of $\gamma$ from $\{\alpha,\beta\}$ |
| $\wedge_\nu\{\gamma \mid \Phi\}$ | |

The meta-linguistic looking expression $\ulcorner\{\alpha,\beta\} \vdash \gamma\urcorner$ usually means that there is a derivation of $\gamma$ from $\{\alpha,\beta\}$. But we abuse notation slightly by using it to denote any witness to this meta-linguistic fact – i.e., an actual (sub)-derivation of $\gamma$ from $\{\alpha,\beta\}$. Otherwise, the above rule makes no sense technically. In practice, $\ulcorner\{\alpha,\beta\} \vdash \gamma\urcorner$ basically means that $\alpha$ and $\beta$ combine to form $\gamma$, which is usually an earlier theorem, or which is simply taken for granted. For example, they are combined by function application. Thus, this rule tells us that an expression $\alpha$ combines with a conjunction $\wedge\{\beta|\Phi\}$ by combining with each conjunct $\beta$.

(3) Item ④ officially reads:

the conjunction of all $\mathbf{V}x$ such that $\mathbf{M}x$

This has type ∧S [=S].

(4) The move from ④ to ⑤ is sanctioned by the following new type-logical rule.

| | |
|---|---|
| $\wedge_\nu\{\Psi \mid \Phi\}$ **/** $\forall\nu\{\Phi\rightarrow\Psi\}$ | [∧-simplification] |

[6] This is not the final form of this rule, which is adjusted later.

Recall that the initial ν is usually omitted when it is obvious which variable is bound, as in the example above, in which case we have the following instance.

$$\wedge\{\mathbf{V}x \mid \mathbf{M}x\} \quad / \quad \forall x\{\mathbf{M}x \rightarrow \mathbf{V}x\}$$

This rule is derivable from basic principles so long as we also posit one further axiom. First, notice the former asserts

$$\text{man}_1 \text{ is } \mathbf{V} \text{ and } \ldots \text{ and } \text{man}_k \text{ is } \mathbf{V}$$

where $\text{man}_1, \ldots, \text{man}_k$ are all the men there are.

The only technical difficulty arises when the latter set is empty, in which case:

$$\wedge\{\mathbf{V}x \mid \mathbf{M}x\} = \wedge\varnothing = ?$$

So, in order to maintain ∧-simplification, we must also postulate:

$$\wedge\varnothing = \mathrm{T}$$

which is admittedly strange.

An alternative is to postulate that $\wedge\varnothing$ = ⦸, in which case 'every man is virtuous' is not true when there are no men; rather, it is *semantically ill-formed*, being a non-starter due to presupposition-failure. This is not such a preposterous idea. But, for the moment, we employ the usual (modern) logical analysis of 'every' according to which 'every F is G' does not assert, or even presuppose, that there are F's.

## 2. every man's mother is kind

<table>
<tr><td>every</td><td>man</td><td>'s</td><td>mother</td><td>[+1]</td><td>is</td><td>kind</td></tr>
<tr><td>$\lambda P_0\wedge\{x \mid Px\}$</td><td>$\mathbf{M}_0$</td><td></td><td></td><td></td><td>$\lambda P_0\{P_1\}$</td><td>$\mathbf{K}_0$</td></tr>
<tr><td colspan="2">$\wedge\{x \mid \mathbf{M}x\}$</td><td>$\lambda x\{x_6\}$</td><td></td><td></td><td colspan="2"></td></tr>
<tr><td colspan="3">$\wedge\{x_6 \mid \mathbf{M}x\}$</td><td>$\lambda x_6\{\mathbf{m}(x)\}$</td><td></td><td colspan="2"></td></tr>
<tr><td colspan="4">$\wedge\{\mathbf{m}(x) \mid \mathbf{M}x\}$</td><td>$\lambda x\{x_1\}$</td><td colspan="2"></td></tr>
<tr><td colspan="5">$\wedge\{\mathbf{m}(x)_1 \mid \mathbf{M}x\}$</td><td colspan="2">$\mathbf{K}_1$</td></tr>
<tr><td colspan="7">$\wedge\{\ \mathbf{K}[\mathbf{m}(x)] \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="7">$\forall x\ \{\ \mathbf{M}x \rightarrow \mathbf{K}[\mathbf{m}(x)]\ \}$</td></tr>
</table>

3. Jay respects every woman

| Jay | [+1] | respects | every | woman | [+2] | |
|---|---|---|---|---|---|---|
| J | $\lambda x\{x_1\}$ | | $\lambda P_0 \wedge \{y \mid Py\}$ | $\mathbf{W}_0$ | | |
| | | | $\wedge\{y \mid \mathbf{W}y\}$ | | $\lambda x\{x_2\}$ | |
| | | $\lambda y_2 \lambda x_1 \mathbf{R}xy$ | $\wedge\{y_2 \mid \mathbf{W}y\}$ | | | |
| $J_1$ | | $\wedge\{\lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\}$ | | | | ① |
| $\wedge\{ \mathbf{R}Jy \mid \mathbf{W}y \}$ | | | | | | ② |
| $\forall y \{ \mathbf{W}y \rightarrow \mathbf{R}Jy \}$ | | | | | | |

**Explanation:**

(1) Composition ① appeals to $\wedge$-composition; in particular, one combines $\lambda y_2 \lambda x_1 \mathbf{R}xy$ with $y_2$ to get $\lambda x_1 \mathbf{R}xy$ inside $\wedge\{\ldots \mid \mathbf{W}y\}$.

(2) Composition ② also appeals to $\wedge$-composition; in particular, one combines $J_1$ with $\lambda x_1 \mathbf{R}xy$ to get $\mathbf{R}Jy$ inside $\wedge\{\ldots \mid \mathbf{W}y\}$.

4. Jay does not respect every woman

| | Jay | [+1] | does-not | respect | every | woman | [+2] | |
|---|---|---|---|---|---|---|---|---|
| | J | $\lambda x\{x_1\}$ | | | $\lambda P_0 \wedge \{y \mid Py\}$ | $\mathbf{W}_0$ | | |
| | | | | | $\wedge\{y \mid \mathbf{W}y\}$ | | $\lambda x\{x_2\}$ | |
| | | | | $\lambda y_2 \lambda x_1 \mathbf{R}xy$ | $\wedge\{y_2 \mid \mathbf{W}y\}$ | | | |
| | | | $\lambda P_1 \lambda x_1 {\sim} Px$ | $\wedge\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$ | | | | |
| | $J_1$ | | $\wedge\{ \lambda x_1 {\sim} \mathbf{R}xy \mid \mathbf{W}y \}$ | | | | | ② |
| | $\wedge\{ {\sim}\mathbf{R}Jy \mid \mathbf{W}y \}$ | | | | | | | |
| ① | ※ $\forall y \{ \mathbf{W}y \rightarrow {\sim}\mathbf{R}xy \}$ ※ | | | | | | | |

As indicated by '※' on line ①, this derivation produces an illegitimate reading. The problem is that 'every woman' has narrow-scope in relation to 'not', but the derivation grants 'every woman' wide-scope.[7]

The obvious question then is how do we formally render quantifier-scope restrictions. We propose to do this by placing restraints on junction-composition, which we do as follows.

[7] We take 'does not respect' as a logical-compound that is different in meaning from the morphological-compound 'dis-respects'. They are interchangeable in some contexts, but not others.

| | |
|---|---|
| $\alpha$ | $\alpha$ is any expression, or null |
| $\wedge_{\nu}\{\beta \mid \Phi\}$ | $\wedge$ ***admits*** $\alpha$ |
| $\alpha,\beta \vdash \gamma$ | a sub-derivation of $\gamma$ from $\{\alpha,\beta\}$ |
| $\wedge_{\nu}\{\gamma \mid \Phi\}$ | |

This leaves the following outstanding theoretical question: how do we specify which items are and are not admitted by $\wedge$? We do so by constructing a list of dis-admitted items.[8] We start the list as follows.[9]

($\wedge_1$) $\wedge$ does not admit $\lambda P_1 \lambda x_1 {\sim} Px$ [does-not].

Now, when we go back and examine the computation above, we notice that composition ② involves $\wedge$ *admitting* ⟦does not⟧, which is now officially prohibited. So, in order to complete the computation, we must instead insert an extra type-logical step, as in the following derivation.

<table>
<tr><td>Jay</td><td>[+1]</td><td>does-not</td><td>respect</td><td>every</td><td>woman</td><td>[+2]</td><td></td></tr>
<tr><td>J</td><td>$\lambda x\{x_1\}$</td><td></td><td></td><td>$\lambda P_0 \wedge\{y \mid Py\}$</td><td>$\mathbf{W}_0$</td><td></td><td></td></tr>
<tr><td colspan="2"></td><td></td><td></td><td colspan="2">$\wedge\{y \mid \mathbf{W}y\}$</td><td>$\lambda x\{x_2\}$</td><td></td></tr>
<tr><td colspan="2"></td><td></td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td colspan="3">$\wedge\{y_2 \mid \mathbf{W}y\}$</td><td></td></tr>
<tr><td colspan="2"></td><td></td><td colspan="4">$\wedge\{\lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\}$</td><td>①</td></tr>
<tr><td colspan="2"></td><td>$\lambda P_1 \lambda x_1 {\sim} Px$</td><td colspan="4">$\lambda x_1 \forall y\{\mathbf{W}y \rightarrow \mathbf{R}xy\}$</td><td>②</td></tr>
<tr><td colspan="2">$J_1$</td><td colspan="5">$\lambda x_1 \, {\sim}\forall y\{\mathbf{W}y \rightarrow \mathbf{R}xy\}$</td><td></td></tr>
<tr><td colspan="7">${\sim}\forall y \{ \mathbf{W}y \rightarrow \mathbf{R}Jy \}$</td><td></td></tr>
</table>

In particular, ② is obtained from ① by the following type-logical derivation.

| | | | | |
|---|---|---|---|---|
| (1) | $\wedge\{\lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\}$ | $\wedge(D_1 \rightarrow S)$ | 1 | Pr |
| (2) | $x_1$ | $D_1$ | 2 | As |
| (3) | $\wedge\{\mathbf{R}xy \mid \mathbf{W}y\}$ | $\wedge S$ [=S] | 12 | 1,2,$\wedge$-com[10] |
| (4) | $\forall y\{\mathbf{W}y \rightarrow \mathbf{R}xy\}$ | S | 12 | $\wedge$-Simp |
| (5) | $\lambda x_1 \forall y\{\mathbf{W}y \rightarrow \mathbf{R}xy\}$ | $D_1 \rightarrow S$ | 1 | 2,4,$\lambda$I |

Moreover, item ② readily combines with ⟦does not⟧ using expanded lambda-conversion.

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[8] We presume that if an item is not explicitly dis-admitted, then it is automatically admitted.

[9] *Ideally*, we will be able to specify what underlying semantic feature, or features, the dis-admitted items share, in virtue of which they are dis-admitted. We might fall short of this ideal, however, in which case we will have to accept a certain amount of theoretical inelegance in our system.

[10] Officially, $\wedge$-com requires a sub-derivation; in most applications, this sub-derivation is an earlier theorem. In this particular case, the earlier theorem is completely trivial – $\{\lambda x_1 Rxy , x_1\} \vdash Rxy$ – and is accordingly not even mentioned. Other times, it is not completely trivial, and is accordingly mentioned, but usually simply as an earlier theorem.

5. every man respects every woman

<table>
<tr><td></td><td>every</td><td>man</td><td>[+1]</td><td>respects</td><td>every</td><td>woman</td><td>[+2]</td></tr>
<tr><td></td><td colspan="3" rowspan="2">$\wedge\{x_1 \mid \mathbf{M}x\}$</td><td>$\lambda y_2\lambda x_1\mathbf{R}xy$</td><td colspan="3">$\wedge\{y_2 \mid \mathbf{W}y\}$</td></tr>
<tr><td colspan="4">$\wedge\{\ \lambda x_1\mathbf{R}xy \mid \mathbf{W}y\ \}$</td></tr>
<tr><td>①</td><td colspan="7">$\wedge\{x_1 \times \wedge\{\ \lambda x_1\mathbf{R}xy \mid \mathbf{W}y\ \} \mid \mathbf{M}x\}$</td></tr>
<tr><td>②</td><td colspan="7">$\wedge\{\ \wedge\{\ \mathbf{R}xy \mid \mathbf{W}y\ \} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td>③</td><td colspan="7">$\wedge\{\ \forall y\{\mathbf{W}y \to \mathbf{R}xy\} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td>④</td><td colspan="7">$\forall x\ \{\ \mathbf{M}x \to \forall y\ \{\mathbf{W}y \to \mathbf{R}xy\}\ \}$</td></tr>
</table>

Composition ① involves combining $x_1$ with $\wedge\{\ \lambda x_1\mathbf{R}xy \mid \mathbf{W}y\ \}$ inside $\wedge\{\ldots \mid \mathbf{M}x\}$, which we do in two steps. First, we produce the shaded material, a product; then we combine the factors of the product by combining $x_1$ and $\lambda x_1\mathbf{R}xy$ inside $\wedge\{\ldots \mid \mathbf{W}y\}$ to produce the shaded material on line ②. Finally, ③ is obtained by applying $\wedge$-simplification to the shaded material in ②, and ④ is obtained by applying $\wedge$-simplification to ③.

The following is an alternative composition. The difference is that ① is obtained by doing the first composition ① inside $\wedge\{\ \ldots \mid \mathbf{W}y\ \}$

<table>
<tr><td></td><td>every</td><td>man</td><td>[+1]</td><td>respects</td><td>every</td><td>woman</td><td>[+2]</td></tr>
<tr><td></td><td colspan="3" rowspan="2">$\wedge\{x_1 \mid \mathbf{M}x\}$</td><td>$\lambda y_2\lambda x_1\mathbf{R}xy$</td><td colspan="3">$\wedge\{y_2 \mid \mathbf{W}y\}$</td></tr>
<tr><td colspan="4">$\wedge\{\ \lambda x_1\mathbf{R}xy \mid \mathbf{W}y\ \}$</td></tr>
<tr><td>①</td><td colspan="7">$\wedge\{\ \lambda x_1\mathbf{R}xy \times \wedge\{x_1 \mid \mathbf{M}x\} \mid \mathbf{W}y\}$</td></tr>
<tr><td>②</td><td colspan="7">$\wedge\{\ \wedge\{\ \mathbf{R}xy \mid \mathbf{M}x\ \} \mid \mathbf{W}y\ \}$</td></tr>
<tr><td>③</td><td colspan="7">$\wedge\{\ \forall x\{\mathbf{M}x \to \mathbf{R}xy\} \mid \mathbf{W}y\ \}$</td></tr>
<tr><td>④</td><td colspan="7">$\forall y\ \{\ \mathbf{W}y \to \forall x\ \{\mathbf{M}x \to \mathbf{R}xy\}\ \}$</td></tr>
</table>

Notice that, although there is an inherent scope-ambiguity, it is harmless in the sense that the two resulting top nodes are logically-equivalent.

6. every man who respects every man's father respects every man's mother

| every | man who respects every man's father | [+1] | respects | every | man | 's | mother | [+2] |
|---|---|---|---|---|---|---|---|---|
| | | | | $\lambda P_0 \wedge\{x \mid Px\}$ | $\mathbf{M}_0$ | | | |
| | | | | $\wedge\{x \mid \mathbf{M}x\}$ | | $\lambda x\{x_6\}$ | | |
| | | | | $\wedge\{x_6 \mid \mathbf{M}x\}$ | | | $\lambda x_6\{\mathbf{m}(x)\}$ | |
| | | | | $\wedge\{\mathbf{m}(x) \mid \mathbf{M}x\}$ | | | | $\lambda x\{x_2\}$ |
| | | | $\lambda y_2 \lambda x_1 \mathbf{R}xy$ | $\wedge\{\mathbf{m}(x)_2 \mid \mathbf{M}x\}$ | | | | |
| $\lambda P_0 \wedge\{x \mid Px\}$ | (insert output of table below) | | | | | | | |
| $\wedge\{ x \mid \mathbf{M}x \;\&\; \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{f}(y)] \} \}$ | | $\lambda x\{x_1\}$ | | | | | | |
| $\wedge\{ x_1 \mid \mathbf{M}x \;\&\; \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{f}(y)] \} \}$ | | | $\wedge\{\lambda x_1 \mathbf{R}[x,\mathbf{m}(y)] \mid \mathbf{M}y\}$ | | | | | |
| $\wedge\{ \wedge\{\mathbf{R}[x,\mathbf{m}(y)] \mid \mathbf{M}y\} \mid \mathbf{M}x \;\&\; \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{f}(y)] \} \}$ | | | | | | | | |
| $\forall x \{ \{\mathbf{M}x \;\&\; \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{f}(y)] \}\} \to \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{m}(y)] \} \}$ | | | | | | | | |

| man | who | [+1] | respects | every | man | 's | father | [+2] | |
|---|---|---|---|---|---|---|---|---|---|
| | $\lambda Q_1 \lambda P_0 \lambda x_0\{Px\&Qx\}$ | $\lambda x\{x_1\}$ | | $\lambda P_0 \wedge\{x \mid Px\}$ | $\mathbf{M}_0$ | | | | |
| | | | | $\wedge\{x \mid \mathbf{M}x\}$ | | $\lambda x\{x_6\}$ | | | |
| | | | | $\wedge\{x_6 \mid \mathbf{M}x\}$ | | | $\lambda x_6\{\mathbf{f}(x)\}$ | | |
| | | | | $\wedge\{ \mathbf{f}(x) \mid \mathbf{M}x \}$ | | | | $\lambda x\{x_2\}$ | |
| | | | $\lambda y_2 \lambda x_1 \mathbf{R}xy$ | $\wedge\{ \mathbf{f}(x)_2 \mid \mathbf{M}x \}$ | | | | | |
| | | | $\wedge\{ \lambda x_1 \mathbf{R}[x,\mathbf{f}(x)] \mid \mathbf{M}x \}$ | | | | | | ① |
| | $\lambda Q_1\, \lambda P_0\, \lambda x_0 \{ Px \;\&\; Qx \}$ | | $\lambda x_1\, \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{f}(y)] \}$ | | | | | | ② |
| $\mathbf{M}_0$ | $\lambda P_0\, \lambda x_0 \{ Px \;\&\; \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{f}(y)] \} \}$ | | | | | | | | |
| $\lambda x_0 \{ \mathbf{M}x \;\&\; \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{f}(y)] \} \}$ | | | | | | | | | |

Note that ① could be combined with ⟦who$_1$⟧, using ∧-composition, *except* that we now add the following further restriction on admissibility.

| | |
|---|---|
| (∧$_2$) | ∧ does not admit any relative-pronoun-phrase.[11] |

This corresponds to a syntactic-restriction on quantifier-movement, according to which ‘every’ cannot be moved outside any relative clause in which it appears.[12]

[11] In other words, an NP headed by a relative pronoun, such as ‘who’ and ‘whose mother’.

Given that ⋀ does not admit relative-pronoun-phrases, we must instead convert ① into ②, using a type-logical transformation, the details of which are left as an exercise.

## 7. 'Some' – Disjunction

In the earlier example,

every man respects every woman

the respective scopes of the two quantifier-phrases is irrelevant to the final computation, since the two resulting formulas are logically equivalent. By contrast, in the following example,

every man respects **some** woman

the scope-ambiguity produces two logically-different readings.

In order to see this, we now offer our new account of 'some', which involves positing a second junction – ⋁, (infinitary) **disjunction** – which is parallel to (infinitary) conjunction. First, we propose a new type-forming operator, defined so that

if $\mathfrak{I}$ is a type, then so is $\bigvee\mathfrak{I}$

As with ⋀, we also add the following expressions to type-theory

$\bigvee_{\nu}\{\varepsilon \mid \Phi\}$

which we read as:

the disjunction [over ν] of all ε such that Φ

As with ⋀, if ε and Φ share exactly one free variable, then ν is omitted, and we have (informal) expressions of the form.

$\bigvee\{\varepsilon \mid \Phi\}$

With this in hand, we interpret 'every' as follows.

| | | |
|---|---|---|
| ⟦some⟧ | = | $\lambda P_0 \bigvee\{x \mid Px\}$ |

For example:

| | | |
|---|---|---|
| ⟦some man⟧ | = | $\bigvee\{x \mid x \text{ is a man}\}$ |
| | = | the disjunction of all men |
| | ≈ | $\text{man}_1$ or $\text{man}_2$ or … or $\text{man}_k$ |

where $\{\text{man}_1, \ldots, \text{man}_k\}$ constitute all the men.

We also add the following simplification principle.

| | |
|---|---|
| $\bigvee_{\nu}\{\Psi \mid \Phi\}$ / $\exists\nu\{\Phi \& \Psi\}$ | [⋁-simplification] |

[12] We now have two items not admitted by ⋀; unfortunately, there is no obvious systematic semantic connection between ⟦does not⟧ and ⟦who⟧. We need to work on this.

# 1. Examples

## 1. Jay does not respect some woman

| Jay | [+1] | does-not | respect | some | woman | [+2] |
|---|---|---|---|---|---|---|
| J | $\lambda x\{x_1\}$ | | | $\lambda P_0 \vee\{y \mid Py\}$ | $\mathbf{W}_0$ | |
| | | | | $\vee\{y \mid \mathbf{W}y\}$ | | $\lambda x\{x_2\}$ |
| | | | $\lambda y_2 \lambda x_1 \mathbf{R}xy$ | $\vee\{y_2 \mid \mathbf{W}y\}$ | | |
| | | $\lambda P_1 \lambda x_1\{\sim Px\}$ | $\vee\{\lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\}$ | | | |
| $J_1$ | | $\vee\{\lambda x_1 \sim \mathbf{R}xy \mid \mathbf{W}y\}$ | | | | |
| $\vee\{\sim \mathbf{R}\text{J}y \mid \mathbf{W}y\}$ | | | | | | |
| $\exists y\{\mathbf{W}y \,\&\, \sim \mathbf{R}xy\}$ | | | | | | |

The computations parallel those for $\wedge$. Rather than repeat these rules, we instead offer the following more general version[13]

| **Junction-Composition (Ж-Com)** | |
|---|---|
| $\alpha$ | $\alpha$ is any expression (**or null**) |
| $Ж_v\{\beta \mid \Phi\}$ | Ж admits $\alpha$ |
| $\{\alpha,\beta\} \vdash \gamma$ | a sub-derivation of $\gamma$ from $\{\alpha,\beta\}$ |
| $Ж_v\{\gamma \mid \Phi\}$ | |

Here, $\alpha$, $\beta$, $\gamma$ are expressions (even junctions[14]), and Ж is any junction.[15]

In the previous derivation, note that $\vee$ admits ⟦does not⟧, and so 'some woman' gains scope over 'does not'. There is an alternative derivation with accords 'does not' wide scope.

[13] This is still not quite the final word on junction-composition.

[14] We use 'junction' in two ways – to refer to the operator, and to refer to the result of applying the operator; this is consonant with the same ambiguity in the word 'conjunction'.

[15] 'Ж' is Cyrillic letter "zhe", which is a soft 'J' (IPA: ʒ); there is no hard 'J' (IPA: ʤ) sound in Russian.

2. Jay does not respect some woman [alternative reading]

<table>
<tr><td>Jay</td><td>[+1]</td><td>does-not</td><td>respect</td><td>some</td><td>woman</td><td>[+2]</td><td></td></tr>
<tr><td>$\mathrm{J}$</td><td>$\lambda x\{x_1\}$</td><td></td><td></td><td>$\lambda P_0 \vee \{y \mid Py\}$</td><td>$\mathbf{W}_0$</td><td></td><td></td></tr>
<tr><td colspan="2"></td><td></td><td></td><td colspan="2">$\vee\{ y \mid \mathbf{W}y \}$</td><td>$\lambda x\{x_2\}$</td><td></td></tr>
<tr><td colspan="2"></td><td></td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td colspan="3">$\vee\{ y_2 \mid \mathbf{W}y \}$</td><td></td></tr>
<tr><td colspan="2"></td><td></td><td colspan="4">$\vee\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$</td><td>①</td></tr>
<tr><td colspan="2"></td><td></td><td colspan="4">$\lambda x_1 \vee\{ \mathbf{R}xy \mid \mathbf{W}y \}$</td><td>②</td></tr>
<tr><td colspan="2"></td><td>$\lambda P_1 \lambda x_1\{\sim Px\}$</td><td colspan="4">$\lambda x_1\ \exists y\{\mathbf{W}y \,\&\, \mathbf{R}xy\}$</td><td>③</td></tr>
<tr><td colspan="2">$\mathrm{J}_1$</td><td colspan="5">$\lambda x_1 \sim\exists y\{\mathbf{W}y \,\&\, \mathbf{R}xy\}$</td><td></td></tr>
<tr><td colspan="7">$\sim\exists y\{\mathbf{W}y \,\&\, \mathbf{R}\mathrm{J}y\}$</td><td></td></tr>
</table>

In this derivation, we perform a type-logical conversion of ① to ② to ③, the final step appealing to ∨-simplification.

3. every man respects some woman

'every man' has wide scope

<table>
<tr><td>every man $_1$</td><td>respects</td><td>some woman $_2$</td></tr>
<tr><td></td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$\vee\{ y_2 \mid \mathbf{W}y \}$</td></tr>
<tr><td>$\wedge\{ x_1 \mid \mathbf{M}x \}$</td><td colspan="2">$\vee\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="3">$\wedge\{ \vee\{ \mathbf{R}xy \mid \mathbf{W}y \} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="3">$\forall x \{ \mathbf{M}x \rightarrow \exists y\{\mathbf{W}y \,\&\, \mathbf{R}xy\} \}$</td></tr>
</table>

'some woman' has wide scope

<table>
<tr><td>every man $_1$</td><td>respects</td><td>some woman $_2$</td></tr>
<tr><td></td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$\vee\{ y_2 \mid \mathbf{W}y \}$</td></tr>
<tr><td>$\wedge\{ x_1 \mid \mathbf{M}x \}$</td><td colspan="2">$\vee\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="3">$\vee\{ \wedge\{ \mathbf{R}xy \mid \mathbf{M}x \} \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="3">$\exists y\{ \mathbf{W}y \,\&\, \forall x \{\mathbf{M}x \rightarrow \mathbf{R}xy\} \}$</td></tr>
</table>

Note, in particular, that the scope-ambiguity is inherent in the rules of composition,[16] so long as both ∧ and ∨ admit each other, which we hereby postulate.[17]

[16] In other words, this is an example of ambiguity that is neither lexical nor structural. The trees have the same parse-structure, and the morphemes have the same meanings.

[17] We do not officially have to postulate this, since our background assumption is that an expression is admitted by a junction unless expressly prohibited.

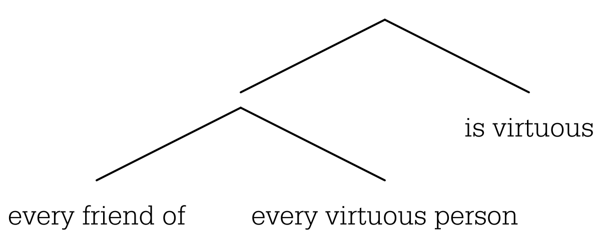
## 8. Another Example of Scope-Ambiguity

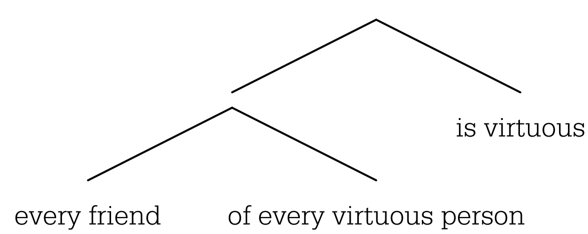
In the previous section we have seen that the rules of junction-composition allow either of two quantifier-phrases to gain scope over the other. The following example involves two occurrences of ‘every’, but is nevertheless, and surprisingly ambiguous with respect to scope.

1. Every friend of every virtuous person is virtuous. [reading 1]

<table>
<tr><th></th><th>every</th><th>friend</th><th>of</th><th>every</th><th>virtuous</th><th>[mod]</th><th>person</th><th>[+1]</th><th>is</th><th>virtuous</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>$\mathbf{V}_0$</td><td>$\lambda Q_0 \lambda P_0 \lambda x_0\{Px \& Qx\}$</td><td></td><td></td><td>$\lambda P_0\{P_1\}$</td><td>$\mathbf{V}_0$</td></tr>
<tr><td></td><td></td><td></td><td></td><td>$\lambda P_0 \wedge\{y \mid Py\}$</td><td colspan="2">$\lambda P_0\ \lambda x_0\ \{\ Px\ \&\ \mathbf{V}x\ \}$</td><td>$\mathbf{P}_0$</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td colspan="3">$\lambda x_0\ \{\ \mathbf{P}x\ \&\ \mathbf{V}x\ \}$</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>$\lambda x\{x_6\}$</td><td colspan="4">$\wedge\{\ y \mid \mathbf{P}y\ \&\ \mathbf{V}y\ \}$</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\lambda y_6 \lambda x_0 \mathbf{F}xy$</td><td colspan="5">$\wedge\{\ y_6 \mid \mathbf{P}y\ \&\ \mathbf{V}y\ \}$</td><td></td><td></td><td></td></tr>
<tr><td></td><td>$\lambda P_0 \wedge\{x \mid Px\}$</td><td colspan="6">$\wedge\{\ \lambda x_0 \mathbf{F}xy \mid \mathbf{P}y\ \&\ \mathbf{V}y\ \}$</td><td></td><td></td><td></td></tr>
<tr><td>①</td><td colspan="7">$\wedge\{\ \lambda P_0 \wedge\{x \mid Px\} \times \lambda x_0 \mathbf{F}xy \mid \mathbf{P}y\ \&\ \mathbf{V}y\ \}$</td><td></td><td></td><td></td></tr>
<tr><td></td><td colspan="7">$\wedge\{\ \wedge\{x \mid \mathbf{F}xy\} \mid \mathbf{P}y\ \&\ \mathbf{V}y\ \}$</td><td>$\lambda x\{x_1\}$</td><td></td><td></td></tr>
<tr><td></td><td colspan="8">$\wedge\{\ \wedge\{\ x_1 \mid \mathbf{F}xy\ \} \mid \mathbf{P}y\ \&\ \mathbf{V}y\ \}$</td><td colspan="2">$\mathbf{V}_1$</td></tr>
<tr><td></td><td colspan="10">$\wedge\{\ \wedge\{\ \mathbf{V}x \mid \mathbf{F}xy\ \} \mid \mathbf{P}y\ \&\ \mathbf{V}y\ \}$</td></tr>
<tr><td></td><td colspan="10">$\forall y\ \{\ \{\mathbf{P}y\ \&\ \mathbf{V}y\} \rightarrow \forall x\{\mathbf{F}xy \rightarrow \mathbf{V}x\}\ \}$</td></tr>
</table>

① is obtained by ⟦every⟧ being absorbed by ⟦friend of …⟧. Note that ⟦every⟧ is anti-tonic (see Appendix 2), so this is an exception to the general rule that ∧ does not admit anti-tonic functions. Note that the following alternative parsings might seem promising




but these don't help, since ⟦every friend of⟧ and ⟦every friend⟧ are *also* anti-tonic (see Appendix 2). So we are faced with ∧ admitting a few anti-tonic functions, even though for the most part ∧ does not admit anti-tonic functions.

2. Every friend of every virtuous person is virtuous. [reading 2]

| every | friend | of | every | virtuous | [mod] | person | [+1] | is | virtuous |
|---|---|---|---|---|---|---|---|---|---|
| | | | | $\mathbf{V}_0$ | $\lambda Q_0 \lambda P_0 \lambda x_0\{Px\&Qx\}$ | | | $\lambda P_0\{P_1\}$ | $\mathbf{V}_0$ |
| | | | | $\lambda P_0\ \lambda x_0\ \{\ Px\ \&\ \mathbf{V}x\ \}$ | | $\mathbf{P}_0$ | | | |
| | | | $\lambda P_0 \wedge \{y \mid Py\}$ | $\lambda x_0\ \{\ \mathbf{P}x\ \&\ \mathbf{V}x\ \}$ | | | | | |
| | | $\lambda x\ \{x_6\}$ | $\wedge\ \{\ y \mid \mathbf{P}y\ \&\ \mathbf{V}y\ \}$ | | | | | | |
| | $\lambda y_6 \lambda x_0\ \mathbf{F}xy$ | $\wedge\ \{\ y_6 \mid \mathbf{P}x\ \&\ \mathbf{V}y\ \}$ | | | | | | | |
| | $\wedge\ \{\ \lambda x_0 \mathbf{F}xy \mid \mathbf{P}y\ \&\ \mathbf{V}y\ \}$ | | | | | | | | |
| $\lambda P_0 \wedge \{x \mid Px\}$ | $\lambda x_0\ \forall y\ \{\ \{\mathbf{P}y\ \&\ \mathbf{V}y\} \to \mathbf{F}xy\ \}$ | | | | | | | | |
| $\wedge\ \{\ x \mid \forall y\ \{\ \{\mathbf{P}y\ \&\ \mathbf{V}y\} \to \mathbf{F}xy\ \}\ \}$ | | | | | | | $\lambda x\ \{x_1\}$ | | |
| $\wedge\ \{\ x_1 \mid \forall y\ \{\ \{\mathbf{P}y\ \&\ \mathbf{V}y\} \to \mathbf{F}xy\ \}\ \}$ | | | | | | | | $\mathbf{V}_1$ | |
| $\wedge\ \{\ \mathbf{V}x \mid \forall y\ \{\ \{\mathbf{P}y\ \&\ \mathbf{V}y\} \to \mathbf{F}xy\ \}\ \}$ | | | | | | | | | |
| $\forall x\ \{\ \forall y\{\{\mathbf{P}y\ \&\ \mathbf{V}y\} \to \mathbf{F}xy\} \to \mathbf{V}x\ \}$ | | | | | | | | | |

This reading is plausible, but if we substitute other genitive nouns, then the reading is completely implausible. For example, the second reading of

every **child** of every virtuous person is virtuous

produces a semantically-impossible antecedent – for suppose *c* is a child of every virtuous person; then *c* is virtuous, and therefore *c* is a child of *c*!

## 9. 'No' [Universal-Negative]

We have redesigned ⟦every⟧ and ⟦some⟧. Next on the list is ⟦no⟧, which we formalize as follows, using the Medieval idea that 'no' is a combination of *universal* and *negative*.

$$[\![\text{no}]\!] \quad = \quad \lambda P_0 \wedge\{\neg \times x \mid Px\}$$

where

$$\neg \quad =_{\text{df}} \quad \lambda P{\sim}P$$

For example,

$$[\![\text{no man}]\!] \quad = \quad \wedge\{\neg \times x \mid \mathbf{M}x\}$$

Accordingly, intuitively, ⟦no man⟧ corresponds to the following conjunction

not-man$_1$ and not-man$_2$ and … and not-man$_k$

where {man$_1$, …, man$_k$} constitute all the men.

1. no man is virtuous

<table>
<tr><td>no</td><td>man</td><td>[+1]</td><td>is</td><td>virtuous</td></tr>
<tr><td>$\lambda P_0 \wedge \{\neg \times x \mid Px\}$</td><td>$\mathbf{M}_0$</td><td rowspan="2">$\lambda x\{x_1\}$</td><td>$\lambda P_0\{P_1\}$</td><td>$\mathbf{V}_0$</td></tr>
<tr><td colspan="2">$\wedge\{ \neg \times x \mid \mathbf{M}x \}$</td><td colspan="2" rowspan="2">$\mathbf{V}_1$</td></tr>
<tr><td colspan="3">$\wedge\{ \neg \times x_1 \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="5">$\wedge\{ \neg \times \mathbf{V}x \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="5">$\wedge\{ \sim\mathbf{V}x \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="5">$\forall x \{ \mathbf{M}x \rightarrow \sim\mathbf{V}x \}$</td></tr>
</table>

2. no man respects every woman

<table>
<tr><td>no</td><td>man</td><td>[+1]</td><td>respects</td><td>every</td><td>woman</td><td>[+2]</td><td></td></tr>
<tr><td>$\lambda P_0 \wedge \{\neg \times x \mid Px\}$</td><td>$\mathbf{M}_0$</td><td rowspan="2">$\lambda x\{x_1\}$</td><td rowspan="3">$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$\lambda P_0 \wedge \{y \mid Py\}$</td><td>$\mathbf{W}_0$</td><td rowspan="2">$\lambda x\{x_2\}$</td><td></td></tr>
<tr><td colspan="2">$\wedge\{\neg \times x \mid \mathbf{M}x \}$</td><td colspan="2">$\wedge\{ y \mid \mathbf{W}y \}$</td><td></td></tr>
<tr><td colspan="3" rowspan="3">$\wedge\{ \neg \times x_1 \mid \mathbf{M}x \}$</td><td colspan="3">$\wedge\{ y_2 \mid \mathbf{W}y \}$</td><td></td></tr>
<tr><td colspan="4">$\wedge\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$</td><td>①</td></tr>
<tr><td colspan="4">$\lambda x_1 \; \forall y\{\mathbf{W}y \rightarrow \mathbf{R}xy\}$</td><td>②</td></tr>
<tr><td colspan="7">$\wedge\{ \neg \times \forall y\{\mathbf{W}y \rightarrow \mathbf{R}xy\} \mid \mathbf{M}x \}$</td><td></td></tr>
<tr><td colspan="7">$\wedge\{ \sim\forall y\{\mathbf{W}y \rightarrow \mathbf{R}xy\} \mid \mathbf{M}x \}$</td><td></td></tr>
<tr><td colspan="7">$\forall x \{ \mathbf{M}x \rightarrow \sim\forall y\{\mathbf{W}y \rightarrow \mathbf{R}xy\} \}$</td><td></td></tr>
</table>

In the above derivation, the derivation of ② from ① side-steps the following further restriction on admissibility.

| | |
|---|---|
| $(\wedge_3)$ | $\wedge$ does not admit $\wedge\neg$ expressions [no P]. |

This restriction theoretically underwrites the intuition that there is no reading of this sentence that accords wide-scope to ‘every woman’.

## 10. 'Any' – Sub-Junction

In the earlier examples,

> Jay does not respect **every** woman
> no man respects **every** woman

'every' cannot gain scope over 'does not' or 'no'. If we want to express these ideas in English, we must instead use the following sentences.

> Jay does not respect **any** woman
> no man respects **any** woman

The natural hypothesis is that 'any' is how we pronounce 'every' when we want it to slip outside another operator. Unfortunately, this hypothesis is defeated by the following example.

> no man respects **any** woman who does not respect **him**

The problem is that 'no man' has to out-scope 'any woman…him' in order to *bind* 'him', so 'any woman…him' cannot simply be moved past 'no man' by quantifier-movement. We will deal with this problem in detail after we reconsider pronoun-binding.[18]

We next observe that the word 'any' is quite eccentric grammatically, in a way similar to 'ever' and 'either'; in particular, they are all *assertorically-deficient*. For example, if I ask

☺ does **any**one have a question?
☺ have you **ever** been to Paris?
☺ do you know **either** of these people?

you are not grammatically-permitted to answer:

☹ yes, **any**one has a question
☹ yes, I have **ever** been to Paris
☹ yes, I know **either** of these people

Also, one can say the following.

☺ **every** student is sitting

but not the following.

☹ **any** student is sitting

On the other hand, one can say **either** of the following.

☺ **every** student caught cheating will be expelled
☺ **any** student caught cheating will be expelled

In the last case, the difference seems to be that the latter, but not the former, carries *modal* (*subjunctive*) *force*. This explains why the following is good or bad, respectively, according to whether it is subjunctive or indicative in character.

---

[18] Another hypothesis, that 'any' corresponds to narrow-scope 'some' is similarly defeated by examples involved with pronoun-binding. See Section 19.

**any** pet of mine is neutered[19]

By way of accounting for the behavior of ‘any’, we propose the following as our overall hypothesis.

> ‘any **A** is B’ is not independently-assertive;
> rather, it is **sub-assertional**.

In order to formalize this hypothesis, we propose yet another junction, Л,[20] called *sub-junction* (short for *sub-conjunction*), with its own special properties, which are summarized as follows.

(1) if $\mathfrak{I}$ is a type, then so is Л$\mathfrak{I}$
(2) ЛS ≠ S
(3) Л *never* simplifies [sub-assertional]
(4) Л admits all expressions, some of which promote Л to ∧ (Л-promotion)

## 11. Examples of ‘any’

1. Jay does not respect any woman

| Jay | [+1] | does-not | respect | any | woman | [+2] | |
|---|---|---|---|---|---|---|---|
| J | $\lambda x\{x_1\}$ | | | $\lambda P_0$Л$\{y \mid Py\}$ | $\mathbf{W}_0$ | | |
| | | | | Л$\{ y \mid \mathbf{W}y \}$ | | $\lambda x\{x_2\}$ | |
| | | | $\lambda y_2 \lambda x_1 \mathbf{R}xy$ | Л$\{ y_2 \mid \mathbf{W}y \}$ | | | |
| | | $\lambda P_1 \lambda x_1\{\sim Px\}$ | Л$\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$ | | | | |
| $J_1$ | | $\wedge\{ \lambda x_1 \sim \mathbf{R}xy \mid \mathbf{W}y \}$ | | | | | ① |
| $\wedge\{ \sim \mathbf{R}Jy \mid \mathbf{W}y \}$ | | | | | | | |
| $\forall y \{ \mathbf{W}y \to \sim \mathbf{R}xy \}$ | | | | | | | |

Note that, unlike ∧, Л admits ⟦does not⟧, which moreover promotes it to ∧, which enables us to treat the resulting phrase as genuinely assertive.

The overall rule is that anti-tonic functions, which are *mostly* the functions not admitted by ∧, are the primary examples of functions that promote Л to ∧.

[19] Actually, we only neuter our *mammalian* pets; the others remain intact!
[20] Cyrillic letter “ehl”, which is short for ‘любой’ [‘liuboi’], which is Russian for (approximately) ‘any one’.

2. no man respects any woman

<table>
<tr><td>no</td><td>man</td><td>[+1]</td><td>respects</td><td>any</td><td>woman</td><td>[+2]</td><td></td></tr>
<tr><td>$\lambda P_0 \wedge\{\neg \times x \mid Px\}$</td><td>$\mathbf{M}_0$</td><td rowspan="2">$\lambda x\{x_1\}$</td><td rowspan="3">$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$\lambda P_0 \text{Л}\{y \mid Py\}$</td><td>$\mathbf{W}_0$</td><td rowspan="2">$\lambda x\{x_2\}$</td><td></td></tr>
<tr><td colspan="2">$\wedge\{ \neg \times x \mid \mathbf{M}x \}$</td><td colspan="2">$\text{Л}\{ y \mid \mathbf{W}y \}$</td><td></td></tr>
<tr><td colspan="3" rowspan="2">$\wedge\{ \neg \times x_1 \mid \mathbf{M}x \}$</td><td colspan="3">$\text{Л}\{ y_2 \mid \mathbf{W}y \}$</td><td></td></tr>
<tr><td colspan="4">$\text{Л}\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$</td><td></td></tr>
<tr><td colspan="7">$\wedge\{ \neg \times \boxed{x_1 \times \text{Л}\{ \lambda x_1 \mathbf{R}xy} \mid \mathbf{W}y \} \mid \mathbf{M}x \}$</td><td>①</td></tr>
<tr><td colspan="7">$\wedge\{ \neg \times \text{Л}\{ \lambda x \mathbf{R}xy \mid \mathbf{W}y \} \mid \mathbf{M}x \}$</td><td>②</td></tr>
<tr><td colspan="7">$\wedge\{ \wedge\{ \sim\mathbf{R}xy \mid \mathbf{W}y \} \mid \mathbf{M}x \}$</td><td>③</td></tr>
<tr><td colspan="7">$\forall x \{ \mathbf{M}x \rightarrow \forall y\{\mathbf{W}y \rightarrow \sim\mathbf{R}xy\} \}$</td><td>④</td></tr>
</table>

This derivation treats 'no man' as having wide scope. This is accomplished on line ① by the ⋀-expression absorbing the Л-expression in the absolutely minimal way. Then ② is obtained from ① by applying junction-combination to the boxed material. Then ③ is obtained from ② by the Л-expression absorbing ¬, noting that ¬ promotes Л to ⋀. Finally, ④ is obtained from ③ by two applications of ⋀-simplification.

The following is an alternative derivation, which grants 'any woman' wide-scope.

<table>
<tr><td>no</td><td>man</td><td>[+1]</td><td>respects</td><td>any</td><td>woman</td><td>[+2]</td><td></td></tr>
<tr><td>$\lambda P_0 \wedge\{\neg \times x \mid Px\}$</td><td>$\mathbf{M}_0$</td><td rowspan="2">$\lambda x\{x_1\}$</td><td rowspan="3">$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$\lambda P_0 \text{Л}\{y \mid Py\}$</td><td>$\mathbf{W}_0$</td><td rowspan="2">$\lambda x\{x_2\}$</td><td></td></tr>
<tr><td colspan="2">$\wedge\{ \neg \times x \mid \mathbf{M}x \}$</td><td colspan="2">$\text{Л}\{ y \mid \mathbf{W}y \}$</td><td></td></tr>
<tr><td colspan="3" rowspan="2">$\wedge\{ \neg \times x_1 \mid \mathbf{M}x \}$</td><td colspan="3">$\text{Л}\{ y_2 \mid \mathbf{W}y \}$</td><td></td></tr>
<tr><td colspan="4">$\text{Л}\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$</td><td></td></tr>
<tr><td colspan="7">$\wedge\{ \wedge\{ \neg \times x_1 \mid \mathbf{M}x \} \times \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$</td><td>①</td></tr>
<tr><td colspan="7">$\wedge\{ \wedge\{ \sim\mathbf{R}xy \mid \mathbf{M}x \} \mid \mathbf{W}y \}$</td><td>②</td></tr>
<tr><td colspan="7">$\forall y \{ \mathbf{W}y \rightarrow \forall x\{\mathbf{M}x \rightarrow \sim\mathbf{R}xy\} \}$</td><td>③</td></tr>
</table>

In this derivation, by way of constructing ①, the Л-expression absorbs ⟦no man⟧, which promotes Л to ⋀. Notice that the two readings are logically equivalent.

So far, we have noted in passing three examples of Л-promotion, which we now officially acknowledge.

| | |
|---|---|
| $(\text{Л}_1)$ | ⟦does- not⟧ promotes Л to ⋀. |

| | |
|---|---|
| $(\text{Л}_2)$ | ⟦not⟧ promotes Л to ⋀. |

| | |
|---|---|
| (Л$_3$) | ⟦no P⟧ promotes Л to ∧. |

To these, we now add the following,

| | |
|---|---|
| (Л$_4$) | ⟦if⟧ promotes Л to ∧. |

which is illustrated in the following example.

3. if Jay respects any woman, Jay respects Kay

<table>
<tr><th>if</th><th>Jay$_1$</th><th>respects</th><th>any</th><th>woman</th><th>[+2]</th><th>Jay respects Kay</th></tr>
<tr><td rowspan="4"></td><td rowspan="3"></td><td rowspan="2"></td><td>$\lambda P_0$ Л$\{ y \mid Py \}$</td><td>$\mathbf{W}_0$</td><td></td><td rowspan="5"></td></tr>
<tr><td colspan="2">Л$\{ y \mid \mathbf{W}y \}$</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td colspan="3">Л$\{ y_2 \mid \mathbf{W}y \}$</td></tr>
<tr><td>J$_1$</td><td colspan="4">Л$\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$</td></tr>
<tr><td>$\lambda P \lambda Q\{P \rightarrow Q\}$</td><td colspan="5">Л$\{ \mathbf{R}\text{J}y \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="6">① $\wedge\{ \lambda Q\{\mathbf{R}\text{J}y \rightarrow Q\} \mid \mathbf{W}y \}$</td><td>$\mathbf{R}$JK</td></tr>
<tr><td colspan="7">$\wedge\{ \mathbf{R}\text{J}y \rightarrow \mathbf{R}\text{JK} \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="7">$\forall x \{ \mathbf{W}x \rightarrow \{\mathbf{R}\text{J}x \rightarrow \mathbf{R}\text{JK}\} \}$</td></tr>
</table>

Contrast the previous example with the following.

4. if Jay respects every woman, Jay respects Kay

<table>
<tr><th>if</th><th>Jay$_1$</th><th>respects</th><th>every</th><th>woman</th><th>[+2]</th><th>Jay respects Kay</th></tr>
<tr><td rowspan="4"></td><td rowspan="3"></td><td rowspan="2"></td><td>$\lambda P_0 \wedge\{y \mid Py\}$</td><td>$\mathbf{W}_0$</td><td></td><td rowspan="5"></td></tr>
<tr><td colspan="2">$\wedge\{ y \mid \mathbf{W}y \}$</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td colspan="3">$\wedge\{ y_2 \mid \mathbf{W}y \}$</td></tr>
<tr><td>J$_1$</td><td colspan="4">$\wedge\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$</td></tr>
<tr><td>$\lambda P \lambda Q\{P \rightarrow Q\}$</td><td colspan="5">$\wedge\{ \mathbf{R}\text{J}y \mid \mathbf{W}y \}$<br>$\forall y\{\mathbf{W}y \rightarrow \mathbf{R}\text{J}y\}$ ①</td></tr>
<tr><td colspan="6">$\lambda Q \{ \forall y\{\mathbf{W}y \rightarrow \mathbf{R}\text{J}y\} \rightarrow Q \}$</td><td>$\mathbf{R}$JK</td></tr>
<tr><td colspan="7">$\forall y\{\mathbf{W}y \rightarrow \mathbf{R}\text{J}y\} \rightarrow \mathbf{R}\text{JK}$</td></tr>
</table>

Note: item ① involves a type-logical transformation, which is required since we have the following further admissibility restriction.

| | |
|---|---|
| (∧$_4$) | ∧ does not admit ⟦if⟧. |

5. no man recommends any woman to any man

| no man [+1] | recommends | any woman [+2] | to any man |
|---|---|---|---|
| | $\lambda y_2 \lambda z_3 \lambda x_1 \mathbf{R}xyz$ | $\text{Л}\{ y_2 \mid \mathbf{W}y \}$ | |
| | $\text{Л}\{ \lambda z_3 \lambda x_1 \mathbf{R}xyz \mid \mathbf{W}y \}$ | | $\text{Л}\{ z_3 \mid \mathbf{M}z \}$ |
| $\wedge\{ \neg \times x_1 \mid \mathbf{M}x \}$ | $\text{Л}\{ \text{Л}\{ \lambda x_1 \mathbf{R}xyz \mid \mathbf{M}z \} \mid \mathbf{W}y \}$ | | |
| $\wedge\{ \neg \times x_1 \times \text{Л}\{ \text{Л}\{ \lambda x_1 \mathbf{R}xyz \mid \mathbf{M}z \} \mid \mathbf{W}y \} \mid \mathbf{M}x \}$ | | | |
| $\wedge\{ \neg \times \text{Л}\{ \text{Л}\{ \mathbf{R}xyz \mid \mathbf{M}z \} \mid \mathbf{W}y \} \mid \mathbf{M}x \}$ | | | |
| $\wedge\{ \wedge\{ \neg \times \text{Л}\{ \mathbf{R}xyz \mid \mathbf{M}z \} \mid \mathbf{W}y \} \mid \mathbf{M}x \}$ | | | |
| $\wedge\{ \wedge\{ \wedge\{ \neg \times \mathbf{R}xyz \mid \mathbf{M}z \} \mid \mathbf{W}y \} \mid \mathbf{M}x \}$ | | | |
| $\wedge\{ \wedge\{ \wedge\{ {\sim}\mathbf{R}xyz \mid \mathbf{M}z \} \mid \mathbf{W}y \} \mid \mathbf{M}\, x \}$ | | | |
| $\forall x\{ \mathbf{M}x \to \forall y\{ \mathbf{W}y \to \forall z\{\mathbf{M}z \to {\sim}\mathbf{R}xyz\} \} \}$ | | | |

Notice, in the computation, that ⟦not⟧ (i.e., $\neg$) penetrates two Л-junctions, promoting each to $\wedge$.

## 12. Anti-Tonic Functions

All examples of any-promotion involve functions that are *monotonic-decreasing* (also called *anti-tonic*). The basic idea is that

a function $\phi$ is monotonic-decreasing if and only if $\forall x \forall y\{x \leqslant y \to \phi(y) \leqslant \phi(x)\}$

Here, $\leqslant$ is an order-relation, which is to say a relation that is reflexive and transitive. The key to applying this idea to semantics is devise a general definition of $\leqslant$ for all type-theoretic items. This is done in Appendix 2.

Moreover, according to this definition, the functions ⟦not⟧, ⟦does not⟧, ⟦if⟧, and ⟦no A⟧ are all monotonic-decreasing. We now propose the following postulate concerning any-promotion.

> if $\alpha$ is monotonic-decreasing, then $\alpha$ promotes Л to $\wedge$.

For the most part the items that promote Л to $\wedge$ are precisely the items that $\wedge$ does not admit. Unfortunately, there are known exceptions

> if $\alpha$ is monotonic-decreasing, then $\wedge$ does not admit $\alpha$.
>
> **known exceptions:**
>
> $\wedge$ admits ⟦every⟧;
> $\wedge$ admits ⟦if $x$ is A⟧.

How to characterize these, and other, exceptions remains a theoretical puzzle.

## 13. The Cyrillic Letter ‘Л’ Sometimes Looks Like ‘∧’

Before continuing, as a side-bar, we note that oftentimes in Russia, the Cyrillic letter ‘Л’ is written more like its Greek ancestor ‘Λ’ (lambda), and hence more like our conjunction sign (‘∧’). In particular, examine the following two signs, one at the tomb of a major communist hero, and the other at a dealership for a highly esteemed Swedish car manufacturer.[21]



## 14. Pronoun-Binding

We now return to pronoun-binding, problems with which set us on this path in the first place. We follow the same basic binding principles as before. In particular, we propose two anaphoric morphemes,

| | |
|---|---|
| ($\alpha$) | creates an anaphoric-role (where $\alpha$ is a negative integer) |
| [$\alpha$] | produces an item to fill (bind) the anaphoric role created by ($\alpha$) |

which are semantically rendered as follows.

$$(\alpha) \quad \curvearrowright \quad \lambda x_\alpha\{x\}$$
$$[\alpha] \quad \curvearrowright \quad \lambda x\{x_\alpha\}$$

We also propose the morpheme ‘e’, which is the essentially-anaphoric pronoun-root, which is semantically vacuous.

$$\text{e} \quad \curvearrowright \quad \varnothing$$

Rather, ‘e’ serves as merely a vehicle/anchor for attaching various markers, including gender, number, case, and alpha-markers. The pronunciation of the various compounds so produced is generally *irregular*. For example:

| | | |
|---|---|---|
| e + masculine + singular + nominative | = | he |
| e + feminine + singular + nominative | = | she |
| e + masculine + singular + genitive | = | his |
| e + feminine + singular + genitive | = | her |

[21] Lenin and Volvo.

## 15. Simple Examples

The following are simple examples of how binding works.

1. Jay respects his mother

<table>
<tr><td>Jay</td><td>[+1]</td><td>[–1]</td><td>respects</td><td>(–1)</td><td colspan="2">his</td><td>mother</td><td>[+2]</td></tr>
<tr><td></td><td>$\lambda x\{x_1\}$</td><td>$\lambda x\{x_{-1}\}$</td><td></td><td></td><td>e+M+S[22]</td><td>[+6]</td><td></td><td></td></tr>
<tr><td>J</td><td colspan="2">$\lambda x\{x_1 \times x_{-1}\}$</td><td></td><td>$\lambda x_{-1}\{x\}$</td><td>$\varnothing$</td><td></td><td></td><td></td></tr>
<tr><td colspan="3"></td><td></td><td colspan="2">$\lambda x_{-1}\{x\}$</td><td>$\lambda x\{x_6\}$</td><td></td><td></td></tr>
<tr><td colspan="3"></td><td></td><td colspan="3">$\lambda x_{-1}\{x_6\}$</td><td>$\lambda x_6\{\mathbf{m}(x)\}$</td><td></td></tr>
<tr><td colspan="3"></td><td></td><td colspan="4">$\lambda x_{-1}\{\mathbf{m}(x)\}$</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td colspan="3"></td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td colspan="5">$\lambda x_{-1}\{\mathbf{m}(x_2)\}$</td></tr>
<tr><td colspan="3">$\mathrm{J}_1 \times \mathrm{J}_{-1}$</td><td colspan="6">$\lambda y_{-1}\ \lambda x_1\ \mathbf{R}[x,\mathbf{m}(y)]$</td></tr>
<tr><td colspan="9">$\mathbf{R}[\mathrm{J},\mathbf{m}(\mathrm{J})]$</td></tr>
</table>

2. Kay's father respects her mother

<table>
<tr><td>Kay</td><td>'s</td><td>[–1]</td><td>father</td><td>[+1]</td><td>respects</td><td>(–1)</td><td colspan="2">her</td><td>mother</td><td>[+2]</td></tr>
<tr><td></td><td>$\lambda x\{x_6\}$</td><td>$\lambda x\{x_{-1}\}$</td><td></td><td></td><td></td><td></td><td>e+F+S</td><td>[+6]</td><td></td><td></td></tr>
<tr><td>K</td><td colspan="2">$\lambda x\{x_6 \times x_{-1}\}$</td><td></td><td></td><td></td><td>$\lambda x_{-1}\{x\}$</td><td>$\varnothing$</td><td>$\lambda x\{x_6\}$</td><td></td><td></td></tr>
<tr><td colspan="3"></td><td></td><td></td><td></td><td colspan="3">$\lambda x_{-1}\{x_6\}$</td><td>$\lambda x_6\{\mathbf{m}(x)\}$</td><td></td></tr>
<tr><td colspan="3">$\mathrm{K}_6 \times \mathrm{K}_{-1}$</td><td>$\lambda x_6\{\mathbf{f}(x)$</td><td></td><td></td><td colspan="4">$\lambda x_{-1}\{\mathbf{m}(x)\}$</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td colspan="4">$\mathbf{f}(\mathrm{K}) \times \mathrm{K}_{-1}$</td><td>$\lambda x\{x_1\}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td colspan="5">$\lambda x_{-1}\{\mathbf{m}(x)_2\}$</td></tr>
<tr><td colspan="5">$\mathbf{f}(\mathrm{K})_1 \times \mathrm{K}_{-1}$</td><td colspan="6">$\lambda y_{-1}\ \lambda x_1\ \mathbf{R}[x,\ \mathbf{m}(y)]$</td></tr>
<tr><td colspan="11">$\mathbf{f}(\mathrm{K})_1 \times \lambda x_1\ \mathbf{R}[x,\ \mathbf{m}(\mathrm{K})]$</td></tr>
<tr><td colspan="11">$\mathbf{R}[\ \mathbf{f}(\mathrm{K}),\ \mathbf{m}(\mathrm{K})\ ]$</td></tr>
</table>

[22] We ignore the semantics of gender and number of the moment, treating both as semantically vacuous.

# 16. Quantifiers and Pronoun-Binding – Original Theory

Pronoun-binding applies equally well to QPs, although we must compute some "gnarly" functions, as in the following examples.

1. every man respects his mother

<table>
<tr><th>every</th><th>man</th><th>[+1]</th><th>[−1]</th><th>respects</th><th>(−1)</th><th colspan="2">his</th><th>mother</th><th>[+2]</th></tr>
<tr><td rowspan="2">$\lambda P_0 \lambda Q \forall x\{Px \rightarrow Qx\}$</td><td rowspan="2">$\mathbf{M}_0$</td><td rowspan="2">$\lambda x\{x_1\}$</td><td rowspan="2">$\lambda x\{x_{-1}\}$</td><td rowspan="5">$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td rowspan="2">$\lambda x_{-1}\{x\}$</td><td>e+M+S</td><td>[+6]</td><td rowspan="3">$\lambda x_6\{\mathbf{m}(x)\}$</td><td rowspan="4">$\lambda x\{x_2\}$</td></tr>
<tr><td>$\varnothing$</td><td>$\lambda x\{x_6\}$</td></tr>
<tr><td colspan="2">$\lambda Q\ \forall x\{\ \mathbf{M}x \rightarrow Qx\ \}$</td><td colspan="2">$\lambda x\{x_1 \times x_{-1}\}$</td><td colspan="3">$\lambda x_{-1}\{x_6\}$</td></tr>
<tr><td colspan="4" rowspan="3">$\lambda Q_{1\text{-}1}\ \forall x\{\ \mathbf{M}x \rightarrow Qxx\ \}$</td><td colspan="4">$\lambda x_{-1}\{\mathbf{m}(x)\}$</td></tr>
<tr><td colspan="5">$\lambda x_{-1}\{\mathbf{m}(x)_2\}$</td></tr>
<tr><td colspan="6">$\lambda y_{-1}\ \lambda x_1\ \mathbf{R}[x,\mathbf{m}(y)]$</td></tr>
<tr><td colspan="10">$\forall x\ \{\ \mathbf{M}x \rightarrow \mathbf{R}[x,\mathbf{m}(x)]\ \}$</td></tr>
</table>

Here, $Q_{1\text{-}1} =_{df} \lambda\langle x_1, y_{-1}\rangle Q\langle x,y\rangle$

which means that the lambda-abstract for ⟦every man [–1,+1]⟧ is quite complicated, although not as complicated as the abstract involved in the following example.

2. every man is happy if he is virtuous

<table>
<tr><th>every man</th><th>[+1]</th><th>[−1]</th><th>is happy</th><th>if</th><th>(−1)</th><th>he [+1]</th><th>is virtuous</th></tr>
<tr><td rowspan="2">$\lambda Q \forall x\{\mathbf{M}x \rightarrow Qx\}$</td><td>$\lambda x\{x_1\}$</td><td>$\lambda x\{x_{-1}\}$</td><td rowspan="3">$\lambda x_1 \mathbf{H}x$</td><td rowspan="3">$\lambda P \lambda Q\{P \rightarrow Q\}$</td><td>$\lambda x_{-1}\{x\}$</td><td>$\lambda x\{x_1\}$</td><td>$\lambda x_1 \mathbf{V}x$</td></tr>
<tr><td colspan="2">$\lambda x\{x_1 \times x_{-1}\}$</td><td colspan="2">$\lambda x_{-1}\{x_1\}$</td><td></td></tr>
<tr><td colspan="3">$\lambda Q_{1\text{-}1}\ \forall x\ \{\ \mathbf{M}x \rightarrow Qx\ \}$</td><td colspan="3">$\lambda x_{-1} \mathbf{V}x$</td></tr>
<tr><td colspan="4">$\lambda \mathcal{Q}_{-1}\ \forall x\ \{\ \mathbf{M}x \rightarrow \mathcal{Q}\langle x,\mathbf{H}x\rangle\ \}$</td><td colspan="4">$\lambda x_{-1}\ \lambda Q\ \{\mathbf{V}x \rightarrow Q\}$</td></tr>
<tr><td colspan="8">$\forall x\ \{\ \mathbf{M}x \rightarrow \{\mathbf{V}x \rightarrow \mathbf{H}x\}\ \}$</td></tr>
</table>

Here, $\mathcal{Q}_{-1} =_{df} \lambda\langle x_{-1}, \Phi\rangle \mathcal{Q}\langle x,\Phi\rangle$

This is a particularly gnarly abstract[23]; it is completely tractable, although it requires hard work.

[23] Dubbed "faux-Q" by Jayme Johnson.

## 17. Quantifiers and Pronoun-Binding – New Theory

Since there are other, more fundamental, problems with our current theory of quantifier/binding, we welcome an account of QPs and binding that doesn't depend on the gnarly functions of the previous section. In particular, note how much simpler the following computations are, which employ our new account of QPs.

1. every man respects his mother

<table>
<tr><td>every</td><td>man</td><td>[+1]</td><td>[−1]</td><td>respects (−1) his mother [+2]</td></tr>
<tr><td>$\lambda P_0 \wedge\{x \mid Px\}$</td><td>$\mathbf{M}_0$</td><td>$\lambda x\{x_1\}$</td><td>$\lambda x\{x_{-1}\}$</td><td rowspan="3">(calculated earlier)<br>$\lambda y_{-1}\ \lambda z_1\ \mathbf{R}[z,\mathbf{m}(y)]$</td></tr>
<tr><td colspan="2">$\wedge\{\ x \mid \mathbf{M}x\ \}$</td><td colspan="2">$\lambda x\{x_1 \times x_{-1}\}$</td></tr>
<tr><td colspan="4">$\wedge\{\ x_1 \times x_{-1} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="5">$\wedge\{x_1 \times x_{-1} \times \lambda y_{-1}\ \lambda z_1\ \mathbf{R}[z,\mathbf{m}(y)] \mid \mathbf{M}x\}$</td></tr>
<tr><td colspan="5">$\wedge\{x_1 \times \lambda z_1\ \mathbf{R}[z,\mathbf{m}(x)] \mid \mathbf{M}x\}$</td></tr>
<tr><td colspan="5">$\wedge\{\ \mathbf{R}[x,\mathbf{m}(x)] \mid \mathbf{M}x\}$</td></tr>
<tr><td colspan="5">$\forall x\ \{\ \mathbf{M}x \rightarrow \mathbf{R}[x,\mathbf{m}(x)]\ \}$</td></tr>
</table>

2. every man is happy if he is virtuous

<table>
<tr><td>every man</td><td>[+1]</td><td>[−1]</td><td>is happy</td><td>if</td><td>(−1)</td><td>he [+1]</td><td>is virtuous</td></tr>
<tr><td rowspan="2">$\wedge\{\ x \mid \mathbf{M}x\ \}$</td><td>$\lambda x\{x_1\}$</td><td>$\lambda x\{x_{-1}\}$</td><td rowspan="3">$\lambda x_1 \mathbf{H}x$</td><td rowspan="3">$\lambda P \lambda Q\{P \rightarrow Q\}$</td><td>$\lambda x_{-1}\{x\}$</td><td>$\lambda x\{x_1\}$</td><td rowspan="2">$\lambda x_1 \mathbf{V}x$</td></tr>
<tr><td colspan="2">$\lambda x\{x_1 \times x_{-1}\}$</td><td colspan="2">$\lambda x_{-1}\{x_1\}$</td></tr>
<tr><td colspan="3">$\wedge\{\ x \times x_{-1} \mid \mathbf{M}x\ \}$</td><td colspan="3">$\lambda x_{-1} \mathbf{V}x$</td></tr>
<tr><td colspan="4">$\wedge\{\ \mathbf{H}x \times x_{-1} \mid \mathbf{M}x\ \}$</td><td colspan="4">$\lambda x_{-1}\ \lambda Q\ \{\mathbf{V}x \rightarrow Q\}$</td></tr>
<tr><td colspan="8">$\wedge\{\ \mathbf{H}x \times x_{-1} \times \lambda x_{-1}\ \lambda Q\{\mathbf{V}x \rightarrow Q\} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="8">$\wedge\{\ \mathbf{H}x \times \lambda Q\{\mathbf{V}x \rightarrow Q\} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="8">$\wedge\{\ \ \{\mathbf{V}x \rightarrow \mathbf{H}x\} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="8">$\forall x\ \{\ \mathbf{M}x \rightarrow \{\mathbf{V}x \rightarrow \mathbf{H}x\}\ \}$</td></tr>
</table>

Note carefully that this involves another example of an anti-tonic function – namely ⟦if (-1)he is virtuous⟧ – that is admitted by ∧; the other one is ⟦every⟧. It remains a puzzle to figure out what underlying principle can account for these, and perhaps other, exceptions to the initial principle that ∧ admits no anti-tonic functions.

## 18. More Difficult Examples

Not only does the new theory make computations involving anaphora much easier, it provides an account of examples that are intractable using the earlier theory. For example, the earlier theory predicts the wrong semantic-value for 'no-if' sentences, such as the following.

1. no man is happy if he is virtuous

| no man | [+1] | [−1] | is happy | if | (−1) | he [+1] | is virtuous |
|---|---|---|---|---|---|---|---|
| | $\lambda x\{x_1\}$ | $\lambda x\{x_{-1}\}$ | | | $\lambda x_{-1}\{x\}$ | $\lambda x\{x_1\}$ | |
| $\lambda Q\forall x\{\mathbf{M}x\to\sim Qx\}$ | $\lambda x\{x_1\times x_{-1}\}$ | | | | $\lambda x_{-1}\{x_1\}$ | | $\lambda x_1\mathbf{V}x$ |
| $\lambda Q_{1\text{-}1}\ \forall x\ \{\ \mathbf{M}x\to\sim Qx\ \}$ | | | $\lambda x_1\mathbf{H}x$ | $\lambda P\lambda Q\{P\to Q\}$ | $\lambda x_{-1}\mathbf{V}x$ | | |
| $\lambda \mathcal{Q}_{-1}\ \forall x\ \{\ \mathbf{M}x\to\sim\mathcal{Q}\langle x,\mathbf{H}x\rangle\ \}$ | | | | $\lambda x_{-1}\ \lambda Q\ \{\mathbf{V}x\to Q\}$ | | | |
| $\forall x\{\ \mathbf{M}x\to\sim\{\mathbf{V}x\to\mathbf{H}x\}\ \}$<br>$\forall x\{\ \mathbf{M}x\to\{\mathbf{V}x\ \&\ \sim\mathbf{H}x\}$<br>※ every man is virtuous but not happy ※ | | | | | | | |

The new theory proposes the following analysis, which is **much better**.

2. no man is happy if he is virtuous

| | no man | [+1] | [−1] | is happy | if | (−1) | he [+1] | is virtuous |
|---|---|---|---|---|---|---|---|---|
| | | $\lambda x\{x_1\}$ | $\lambda x\{x_{-1}\}$ | | | $\lambda x_{-1}\{x\}$ | $\lambda x\{x_1\}$ | |
| | $\wedge\{\ \neg\times x \mid \mathbf{M}x\ \}$ | $\lambda x\{x_1\times x_{-1}\}$ | | | | $\lambda x_{-1}\{x_1\}$ | | $\lambda x_1\mathbf{V}x$ |
| | $\wedge\{\ \neg\times x_1\times x_{-1} \mid \mathbf{M}x\ \}$ | | | $\lambda x_1\mathbf{H}x$ | $\lambda P\lambda Q\{P\to Q\}$ | $\lambda x_{-1}\mathbf{V}x$ | | |
| ① | $\wedge\{\ \neg\times\mathbf{H}x\times x_{-1} \mid \mathbf{M}x\ \}$ | | | | $\lambda x_{-1}\ \lambda Q\ \{\mathbf{V}x\to Q\}$ | | | |
| ② | $\wedge\{\ \sim\mathbf{H}x\times x_{-1} \mid \mathbf{M}x\ \}$ | | | | | | | |
| | $\wedge\{\ \sim\mathbf{H}x\times\lambda Q\ \{\mathbf{V}x\to Q\} \mid \mathbf{M}x\ \}$ | | | | | | | |
| | $\wedge\{\ \mathbf{V}x\to\sim\mathbf{H}x \mid \mathbf{M}x\ \}$ | | | | | | | |
| | $\forall x\ \{\ \mathbf{M}x\to\{\mathbf{V}x\to\sim\mathbf{H}x\}\ \}$ | | | | | | | |

Notice that ② derives from ① by applying ¬ to $\mathbf{H}x$ in the obvious way. This is moreover not an optional move, in light of the following **forced-locality principle**, which we now make official.

> a product $\alpha\times\beta$ created by a multiplicative-morpheme[24]
> must be (immediately) simplified
> by function-application
> if either factor is an argument for the other

[24] In other words, a morpheme whose interpretation involves ×, which includes 'no' and 'self'.

If we don't institute this principle, we run into numerous problems with functors gaining too-wide scope.[25] In this particular example, without forced locality, we have the following derivation, which is precisely what we are trying to avoid.

<table>
<tr><td>no man [−1] is happy</td><td>if (−1) he is virtuous</td></tr>
<tr><td>$\wedge\{\, \neg \times \mathbf{H}x \times x_{-1} \mid \mathbf{M}x \,\}$</td><td>$\lambda x_{-1}\, \lambda Q\, \{\mathbf{V}x \rightarrow Q\}$</td></tr>
<tr><td colspan="2">$\wedge\{\, \neg \times \mathbf{H}x \times \lambda Q\, \{\mathbf{V}x \rightarrow Q\} \mid \mathbf{M}x \,\}$</td></tr>
<tr><td colspan="2">$\wedge\{\, \neg \times \{\mathbf{V}x \rightarrow \mathbf{H}x\} \mid \mathbf{M}x \,\}$</td></tr>
<tr><td colspan="2">$\wedge\{\, \sim\{\mathbf{V}x \rightarrow \mathbf{H}x\} \mid \mathbf{M}x \,\}$</td></tr>
<tr><td colspan="2">※ $\forall x\, \{\, \mathbf{M}x \rightarrow \sim\{\mathbf{V}x \rightarrow \mathbf{H}x\} \,\}$ ※</td></tr>
</table>

3. every man's mother respects his father

<table>
<tr><td>every man</td><td>'s [−1]</td><td>mother</td><td>[+1]</td><td>respects</td><td>(−1)</td><td>he</td><td>'s</td><td>father</td><td>[+2]</td></tr>
<tr><td>$\wedge\{\, x \mid \mathbf{M}x \,\}$</td><td>$\lambda x\{x_6 \times x_{-1}\}$</td><td></td><td rowspan="2"></td><td rowspan="3"></td><td>$\lambda x_{-1}\{x\}$</td><td>$\varnothing$</td><td>$\lambda x\{x_6\}$</td><td></td><td rowspan="2"></td></tr>
<tr><td colspan="2">$\wedge\{\, x_6 \times x_{-1} \mid \mathbf{M}x \,\}$</td><td>$\lambda x_6\{\mathbf{m}(x)\}$</td><td colspan="3">$\lambda x_{-1}\{x_6\}$</td><td>$\lambda x_6\{\mathbf{f}(x)$</td></tr>
<tr><td colspan="3">$\wedge\{\, \mathbf{m}(x) \times x_{-1} \mid \mathbf{M}x \,\}$</td><td>$\lambda x\{x_1\}$</td><td colspan="4">$\lambda x_{-1}\{\mathbf{f}(x)\}$</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td colspan="4" rowspan="2">$\wedge\{\, \mathbf{m}(x)_1 \times x_{-1} \mid \mathbf{M}x \,\}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td colspan="5">$\lambda x_{-1}\{\mathbf{f}(x)_2\}$</td></tr>
<tr><td colspan="6">$\lambda y_{-1}\, \lambda z_1\, \mathbf{R}[z,\mathbf{f}(y)]$</td></tr>
<tr><td colspan="10">$\wedge\{\, \mathbf{m}(x)_1 \times x_{-1} \times \lambda y_{-1}\, \lambda z_1\, \mathbf{R}[z,\mathbf{f}(y)] \mid \mathbf{M}x \,\}$</td></tr>
<tr><td colspan="10">$\wedge\{\, \mathbf{m}(x)_1 \times \lambda z_1\, \mathbf{R}[z,\mathbf{f}(x)] \mid \mathbf{M}x \,\}$</td></tr>
<tr><td colspan="10">$\wedge\{\, \mathbf{R}[\mathbf{m}(x), \mathbf{f}(x)] \mid \mathbf{M}x \,\}$</td></tr>
<tr><td colspan="10">$\forall x\, \{\, \mathbf{M}x \rightarrow \mathbf{R}[\mathbf{m}(x), \mathbf{f}(x)] \,\}$</td></tr>
</table>

[25] For example, without forced locality, the reflexive morpheme 'self' can attach to *any* earlier verb, when in fact it *should* only attach to the nearest verb. This problem was first discovered by Peter Marchetto.

4. no man's mother respects her father

<table>
<tr><td>no man</td><td>'s</td><td>mother</td><td>[+1, –1]</td><td>respects</td><td>(–1)</td><td colspan="2">her</td><td>father</td><td>[+2]</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td>e+F</td><td>[+6]</td><td></td><td></td></tr>
<tr><td>$\wedge\{\neg \times x \mid \mathbf{M}x\}$</td><td>$\lambda x\{x_6\}$</td><td></td><td></td><td></td><td>$\lambda x_{-1}\{x\}$</td><td>$\varnothing$</td><td>$\lambda x\{x_6\}$</td><td></td><td></td></tr>
<tr><td colspan="2">$\wedge\{ \neg \times x_6 \mid \mathbf{M}x \}$</td><td>$\lambda x_6\{\mathbf{m}(x)\}$</td><td></td><td></td><td colspan="3">$\lambda x_{-1}\{x_6\}$</td><td>$\lambda x\{\mathbf{f}(x)\}$</td><td></td></tr>
<tr><td colspan="3">$\wedge\{ \neg \times \mathbf{m}(x) \mid \mathbf{M}x \}$</td><td>$\lambda x\{x_1 \times x_{-1}\}$</td><td></td><td colspan="4">$\lambda x_{-1}\{\mathbf{f}(x)\}$</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td colspan="4"></td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td colspan="5">$\lambda x_{-1}\{\mathbf{f}(x)_2\}$</td></tr>
<tr><td colspan="4">$\wedge\{ \neg \times \mathbf{m}(x)_1 \times \mathbf{m}(x)_{-1} \mid \mathbf{M}x \}$</td><td colspan="6">$\lambda y_{-1}\ \lambda z_1\ \mathbf{R}[z,\mathbf{f}(y)]$</td></tr>
<tr><td colspan="10">$\wedge\{ \neg \times \mathbf{m}(x)_1 \times \mathbf{m}(x)_{-1} \times \lambda y_{-1}\ \lambda z_1\ \mathbf{R}[z,\mathbf{f}(y)] \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="10">$\wedge\{ \neg \times \mathbf{m}(x)_1 \times \lambda z_1\ \mathbf{R}[z,\mathbf{f}(\mathbf{m}(x))] \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="10">$\wedge\{ \neg \times \mathbf{R}[z,\mathbf{f}(\mathbf{m}(x))] \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="10">$\wedge\{ \sim\mathbf{R}[\mathbf{m}(x), \mathbf{f}(\mathbf{m}(x))] \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="10">$\forall x \{ \mathbf{M}x \rightarrow \sim\mathbf{R}[\mathbf{m}(x), \mathbf{f}(\mathbf{m}(x))] \}$</td></tr>
</table>

## 19. Examples involving ‘any’

Our original theory of binding also cannot account for ‘any’, as in the following examples.

1. no man respects any enemy of his

<table>
<tr><td>no man</td><td>[+1, –1]</td><td>respects</td><td>any</td><td>enemy</td><td>of</td><td>(–1)</td><td>his[26]</td><td>[+2]</td></tr>
<tr><td>$\wedge\{\neg \times x \mid \mathbf{M}x\}$</td><td>$\lambda x\{x_1 \times x_{-1}\}$</td><td></td><td></td><td></td><td>$\lambda x\{x_6\}$</td><td>$\lambda x_{-1}\{x\}$</td><td>$\varnothing$</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>$\lambda z_6 \lambda y_0 \mathbf{E}yz$</td><td colspan="3">$\lambda z_{-1}\{z_6\}$</td><td></td></tr>
<tr><td></td><td></td><td></td><td>$\lambda P_0$Л$\{y \mid Py\}$</td><td colspan="4">$\lambda z_{-1}\ \lambda y_0\ \mathbf{E}yz$</td><td></td></tr>
<tr><td></td><td></td><td></td><td colspan="5">$\lambda z_{-1}$ Л$\{ y \mid \mathbf{E}yz \}$</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td colspan="2"></td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td colspan="6">$\lambda z_{-1}$ Л$\{ y_2 \mid \mathbf{E}yz \}$</td></tr>
<tr><td colspan="2">$\wedge\{\neg \times x_1 \times x_{-1} \mid \mathbf{M}x\}$</td><td colspan="7">$\lambda z_{-1}$ Л$\{ \lambda w_1 \mathbf{R}wy \mid \mathbf{E}yz \}$</td></tr>
<tr><td colspan="9">$\wedge\{\neg \times x_1 \times$ Л$\{ \lambda w_1 \mathbf{R}wy \mid \mathbf{E}yx \} \mid \mathbf{M}x\}$</td></tr>
<tr><td colspan="9">$\wedge\{\neg \times$ Л$_y\{ \mathbf{R}xy \mid \mathbf{E}yx \} \mid \mathbf{M}x\}$</td></tr>
<tr><td colspan="9">$\wedge\{ \wedge_y\{ \sim\mathbf{R}xy \mid \mathbf{E}yx \} \mid \mathbf{M}x\}$</td></tr>
<tr><td colspan="9">$\forall x \{ \mathbf{M}x \rightarrow \forall y\{\mathbf{E}yx \rightarrow \sim\mathbf{R}xy\} \}$</td></tr>
</table>

[26] This is an oddity – some NPs are double-marked for genitive; we ignore the second marking.

2. no man respects any woman who does not respect him

| no man [+1, −1] | respects | any [+2] | woman | who [+1] | does-not | respect | (−1) | him | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | e+M | [+2] |
| | | | | | | $\lambda y_2 \lambda x_1$ $\mathbf{R}xy$ | $\lambda x_{-1}\{x\}$ | $\varnothing$ | $\lambda x\{x_2\}$ |
| | | | | | | | $\lambda x_{-1}\{x_2\}$ | | |
| | | | | | $\lambda P_1 \lambda x_1 {\sim} Px$ | $\lambda y_{-1}\, \lambda x_1\, \mathbf{R}xy$ | | | |
| | | | | $\lambda Q_1 \lambda P_0 \lambda x_0$ $\{Px \& Qx\}$ | $\lambda y_{-1}\, \lambda x_1\, {\sim}\mathbf{R}xy$ | | | | |
| | | | $\mathbf{W}_0$ | $\lambda y_{-1}\, \lambda P_0\, \lambda x_0\, \{ Px \;\&\; {\sim}\mathbf{R}xy \}$ | | | | | |
| | | $\lambda P_0 \text{Л}$ $\{z_2 \mid Pz\}$ | $\lambda y_{-1}\, \lambda x_0\, \{ \mathbf{W}x \;\&\; {\sim}\mathbf{R}xy \}$ | | | | | | |
| | $\lambda y_2 \lambda x_1$ $\mathbf{R}xy$ | $\lambda y_{-1}\, \text{Л}\{ z_2 \mid \mathbf{W}z \;\&\; {\sim}\mathbf{R}zy \}$ | | | | | | | |
| $\wedge\{\neg \times x_1 \times x_{-1} \mid \mathbf{M}x\}$ | $\lambda y_{-1}\, \text{Л}\{ \lambda w_1 \mathbf{R}wz \mid \mathbf{W}z \;\&\; {\sim}\mathbf{R}zy \}$ | | | | | | | | |
| $\wedge\{ \neg \times x_1 \times x_{-1} \times \lambda y_{-1}\, \text{Л}\{ \lambda w_1 \mathbf{R}wz \mid \mathbf{W}z \;\&\; {\sim}\mathbf{R}zy \} \mid \mathbf{M}x \}$ | | | | | | | | | |
| $\wedge\{ \neg \times x_1 \times \text{Л}\{ \lambda w_1 \mathbf{R}wz \mid \mathbf{W}z \;\&\; {\sim}\mathbf{R}zx \} \mid \mathbf{M}x \}$ | | | | | | | | | |
| $\wedge\{ \neg \times \text{Л}_z\{ \mathbf{R}xz \mid \mathbf{W}z \;\&\; {\sim}\mathbf{R}zx \} \mid \mathbf{M}x \}$ | | | | | | | | | |
| $\wedge\{ \wedge_z\{ {\sim}\mathbf{R}xz \mid \mathbf{W}z \;\&\; {\sim}\mathbf{R}zx \} \mid \mathbf{M}x \}$ | | | | | | | | | |
| $\forall x\, \{ \mathbf{M}x \rightarrow \forall z\, \{ (\mathbf{W}z \;\&\; {\sim}\mathbf{R}zx) \rightarrow {\sim}\mathbf{R}xz \} \}$ | | | | | | | | | |

These examples demonstrate that ‘any’ cannot simply be *given* wide scope, for then the ‘any’ phrase is not bound by ‘no man’, but the ‘any’ phrase contains a pronoun that is anaphoric to ‘no man’. The same issue arises in the following more complicated example, in which there are two occurrences of ‘any’.

Another natural hypothesis, which many intro logic students gravitate towards, is that ‘any’ is a narrow-scope ‘some’. This hypothesis is a bit strained, but it accounts for the two examples above. For example, we have the following alternative derivation for the second example.

| no man [+1, −1] | respects | any [+2] | woman who does-not respect (−1) him |
|---|---|---|---|
| | | $\lambda P_0 \vee\{z_2 \mid Pz\}$ | $\lambda y_{-1}\, \lambda x_0\, \{ \mathbf{W}x \;\&\; {\sim}\mathbf{R}xy \}$ |
| | $\lambda y_2 \lambda x_1 \mathbf{R}xy$ | $\lambda y_{-1} \vee\{ \lambda w_1 \mathbf{R}wz \mid \mathbf{W}z \;\&\; {\sim}\mathbf{R}zy \}$ | |
| | $\lambda y_{-1} \vee\{ \lambda w_1 \mathbf{R}wz \mid \mathbf{W}z \;\&\; {\sim}\mathbf{R}zy \}$ | | |
| $\wedge\{\neg \times x_1 \times x_{-1} \mid \mathbf{M}x\}$ | $\lambda y_{-1}\, \lambda w_1\, \exists z\{ \mathbf{W}z \;\&\; {\sim}\mathbf{R}zy \;\&\; \mathbf{R}wz \}$ | | |
| $\wedge\{ {\sim}\exists z\{ \mathbf{W}z \;\&\; {\sim}\mathbf{R}zx \;\&\; \mathbf{R}xz \} \mid \mathbf{M}x \}$ | | | |
| $\forall x\, \{ \mathbf{M}x \rightarrow {\sim}\exists z\{ \mathbf{W}z \;\&\; {\sim}\mathbf{R}zx \;\&\; \mathbf{R}xz \} \}$ | | | |

On the other hand, this approach cannot handle the following example.

3. no man respects any woman if she does not respect him

| no man [+1, −1] | respects | any woman [+2, −2] | if | (−2) she [+1] | does–not | respect | (−1) him [+2] |
|---|---|---|---|---|---|---|---|
| $\wedge\{ x_1 \times x_{-1} \mid \mathbf{M}x \}$ | $\lambda y_2\lambda x_1\mathbf{R}xy$ | $\vee\{ y_2 \times y_{-2} \mid \mathbf{W}y \}$ | | | | $\lambda y_2\lambda x_1\mathbf{R}xy$ | $\lambda v_{-1}\{v_2\}$ |
| | $\vee\{ \lambda x_1\mathbf{R}xy \times y_{-2} \mid \mathbf{W}y \}$ | | | | $\lambda P_1\lambda x_1{\sim}Px$ | $\lambda v_{-1}\lambda x_1\mathbf{R}xy$ | |
| | | | | $\lambda w_{-2}\{w_1\}$ | $\lambda v_{-1}\, \lambda x_1\, {\sim}\mathbf{R}xv$ | | |
| | | | $\lambda P\lambda Q\{P{\rightarrow}Q\}$ | $\lambda v_{-1}\ \lambda w_{-2}\ {\sim}\mathbf{R}wv$ | | | |
| $\wedge\{ \vee\{ \mathbf{R}xy \times y_{-2} \mid \mathbf{W}y \} \times x_{-1} \mid \mathbf{M}x \}$ | | | $\lambda v_{-1}\ \lambda w_{-2}\, \lambda P\{{\sim}\mathbf{R}wv \rightarrow Q\}$ | | | | |
| $\wedge\{ \vee\{ \mathbf{R}xy \times y_{-2} \mid \mathbf{W}y \} \times x_{-1} \times \lambda v_{-1}\ \lambda w_{-2}\, \lambda P\{{\sim}\mathbf{R}wv \rightarrow Q\} \mid \mathbf{M}x \}$ | | | | | | | |
| $\wedge\{ \vee\{ \mathbf{R}xy \times y_{-2} \mid \mathbf{W}y \} \times \lambda w_{-2}\, \lambda P\{{\sim}\mathbf{R}wx \rightarrow Q\} \mid \mathbf{M}x \}$ | | | | | | | |
| $\wedge\{ \vee\{ \mathbf{R}xy \times y_{-2}\ \times \lambda w_{-2}\, \lambda P\{{\sim}\mathbf{R}wx \rightarrow Q\} \mid \mathbf{W}y \} \mid \mathbf{M}x \}$ | | | | | | | |
| $\wedge\{ \vee\{ \mathbf{R}xy \times \lambda P\{{\sim}\mathbf{R}yx \rightarrow Q\} \mid \mathbf{W}y \} \mid \mathbf{M}x \}$ | | | | | | | |
| $\wedge\{ \vee\{{\sim}\mathbf{R}yx \rightarrow\ \mathbf{R}xy \mid \mathbf{W}y \} \mid \mathbf{M}x \}$ | | | | | | | |
| ※ $\forall x \{ \mathbf{M}x \rightarrow \exists y \{ \mathbf{W}y \,\&\, ({\sim}\mathbf{R}yx \times \mathbf{R}xy) \} \}$ ※ | | | | | | | |

The following is another example of how ‘no’ interacts with two occurrences of ‘any’.

4. no man recommends any friend of his to any woman

| no man [+1, −1] | recommends | any [+2] | friend | of | (−1) | his | to | any woman |
|---|---|---|---|---|---|---|---|---|
| | | | | | $\lambda w_{-1}\{w\}$ | $\varnothing$ | $\lambda x\{x_3\}$ | $\text{Л}\{ z \mid \mathbf{W}z \}$ |
| | | | $\lambda z_6\lambda y_0\ \mathbf{F}yz$ | $\lambda x\{x_6\}$ | $\lambda w_{-1}\{w\}$ | | | |
| | | | | $\lambda w_{-1}\{w_6\}$ | | | | |
| | | $\lambda P_0\text{Л}\{y_2 \mid Py\}$ | $\lambda w_{-1}\lambda y_0\mathbf{F}yw$ | | | | | |
| | $\lambda y_2\lambda z_3\lambda x_1\ \mathbf{R}xyz$ | $\lambda w_{-1}\ \text{Л}\{ y_2 \mid \mathbf{F}yw \}$ | | | | | | |
| | $\lambda w_{-1}\ \text{Л}\{ \lambda z_3\lambda x_1\mathbf{R}xyz \mid \mathbf{F}yw \}$ | | | | | | $\text{Л}\{ z_3 \mid \mathbf{W}z \}$ | |
| $\wedge\{\neg \times x_1 \times x_{-1} \mid \mathbf{M}x\}$ | $\lambda w_{-1}\ \text{Л}\{ \text{Л}\{ \lambda x_1\mathbf{R}xyz \mid \mathbf{W}z \} \mid \mathbf{F}yw \}$ | | | | | | | |
| $\wedge\{ \neg \times x_1 \times x_{-1} \times \lambda w_{-1}\ \text{Л}\{ \text{Л}\{ \lambda x_1\mathbf{R}xyz \mid \mathbf{W}z \} \mid \mathbf{F}yw \} \mid \mathbf{M}x\}$ | | | | | | | | |
| $\wedge\{ \neg \times x_1 \times \text{Л}\{ \text{Л}\{ \lambda x_1\mathbf{R}xyz \mid \mathbf{W}z \} \mid \mathbf{F}yx \} \mid \mathbf{M}x\}$ | | | | | | | | |
| $\wedge\{ \neg \times \text{Л}\{ \text{Л}\{ \mathbf{R}xyz \mid \mathbf{W}z \} \mid \mathbf{F}yx \} \mid \mathbf{M}x\}$ | | | | | | | | |
| $\wedge\{ \wedge\{ \wedge\{ {\sim}\mathbf{R}xyz \mid \mathbf{W}z \} \mid \mathbf{F}yx \} \mid \mathbf{M}x\}$ | | | | | | | | |
| $\forall x \{ \mathbf{M}x \rightarrow \forall y \{ \mathbf{F}yx \rightarrow \forall z\{\mathbf{W}z \rightarrow {\sim}\mathbf{R}xyz\} \} \}$ | | | | | | | | |

Notice that $\neg$ crosses two Л-operators, promoting each to $\wedge$.

The following further illustrates how ‘if’ and ‘any’ interact.

5. if Jay respects any woman, then she respects him

<table>
<tr><td>if</td><td>Jay [+1,–1]</td><td>respects</td><td>any woman [+2, –2]</td><td>then</td><td>(–2)</td><td colspan="2">she</td><td>respects</td><td>(–1)</td><td colspan="2">him</td></tr>
<tr><td rowspan="4">$\lambda P \lambda Q \{P \rightarrow Q\}$</td><td rowspan="3">$J_1 \times J_{-1}$</td><td rowspan="2">$\lambda y_2 \lambda x_1$ $\mathbf{R}xy$</td><td rowspan="2">Л$\{y_2 \times y_{-2} \mid \mathbf{W}y\}$</td><td rowspan="4">∅</td><td></td><td>e+F</td><td>[+1]</td><td rowspan="3">$\lambda y_2 \lambda x_1$ $\mathbf{R}xy$</td><td></td><td>e+M</td><td>[+2]</td></tr>
<tr><td>$\lambda z_{-2}\{z\}$</td><td>∅</td><td>$\lambda x\{x_1\}$</td><td>$\lambda w_{-1}\{w\}$</td><td>∅</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td colspan="2">Л$\{ \lambda x_1 \mathbf{R}xy \times y_{-2} \mid \mathbf{W}y \}$</td><td rowspan="2" colspan="3">$\lambda z_{-2}\{z_1\}$</td><td colspan="3">$\lambda w_{-1}\{w_2\}$</td></tr>
<tr><td colspan="3">Л$\{ \mathbf{R}Jy \times J_{-1} \times y_{-2} \mid \mathbf{W}y \}$</td><td colspan="4">$\lambda w_{-1} \lambda x_1 \mathbf{R}xw$</td></tr>
<tr><td colspan="4">$\wedge\{ \lambda Q\{\mathbf{R}Jy \rightarrow Q\} \times J_{-1} \times y_{-2} \mid \mathbf{W}y \}$</td><td colspan="8">$\lambda w_{-1}\, \lambda z_{-2}\, \mathbf{R}zw$</td></tr>
<tr><td colspan="12">$\wedge\{ \lambda Q\{\mathbf{R}Jy \rightarrow Q\} \times J_{-1} \times y_{-2} \times \lambda w_{-1}\, \lambda z_{-1}\, \mathbf{R}zw \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="12">$\wedge\{ \lambda Q\{\mathbf{R}Jy \rightarrow Q\} \times \mathbf{R}yJ \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="12">$\wedge\{ \mathbf{R}Jy \rightarrow \mathbf{R}yJ \mid \mathbf{W}y \}$</td></tr>
<tr><td colspan="12">$\forall y \{ \mathbf{W}y \rightarrow \{ \mathbf{R}Jy \rightarrow \mathbf{R}yJ \} \}$</td></tr>
</table>

## 20. Failed Binding

A theory of binding should explain both positive and negative cases of binding. In this connection, contrast the two previous examples with the following four examples.

1. if every man is virtuous, then he is happy

<table>
<tr><td>if</td><td>every man [+1, –1]</td><td>is virtuous</td><td>then</td><td>(–1) he [+1]</td><td>is happy</td></tr>
<tr><td rowspan="2">$\lambda P \lambda Q\{P \rightarrow Q\}$</td><td>$\wedge\{x_1 \times x_{-1} \mid \mathbf{M}x\}$</td><td>$\lambda x_1 \mathbf{V}x$</td><td>∅</td><td>$\lambda x_{-1}\{x_1\}$</td><td>$\lambda x_1 \mathbf{H}x$</td></tr>
<tr><td colspan="2">$\wedge\{ \mathbf{V}x \times x_{-1} \mid \mathbf{M}x \}$</td><td rowspan="2" colspan="3">$\lambda x_{-1} \mathbf{H}x$</td></tr>
<tr><td colspan="3">※</td></tr>
<tr><td colspan="6">※</td></tr>
</table>

There is no sensible way of combine ⟦if⟧ with $\wedge\{\mathbf{V}x \times x_{-1} \mid \mathbf{M}x\}$; the latter is not a sentence, and $\wedge$ does not admit ⟦if⟧.

The following examples are similar in that ⟦if⟧ cannot be combined with the given antecedent phrase, because it contains an anaphoric-binder [–1] and does not admit ⟦if⟧.

2. if no man is virtuous, then he is happy

| if | no man [+1, −1] | is virtuous | then | (−1) he [+1] | is happy |
|---|---|---|---|---|---|
| | $\wedge\{\neg \times x_1 \times x_{-1} \mid \mathbf{M}x\}$ | $\lambda x_1 \mathbf{V}x$ | $\varnothing$ | $\lambda x_{-1}\{x_1\}$ | $\lambda x_1 \mathbf{H}x$ |
| $\lambda P \lambda Q\{P \to Q\}$ | $\wedge\{ \sim\mathbf{V}x \times x_{-1} \mid \mathbf{M}x \}$ | | | | |
| ※ | | | $\lambda x_{-1}\mathbf{H}x$ | | |
| ※ | | | | | |

3. if Jay respects every woman, then she respects him

| if | Jay [+1, −1] | respects | every woman [+2,−2] | then | (−2) she [+1] | respects | (−1) him [+2] |
|---|---|---|---|---|---|---|---|
| | | $\lambda y_2 \lambda x_1$ $\mathbf{R}xy$ | $\wedge\{ y_2 \times y_{-2} \mid \mathbf{W}y \}$ | | | $\lambda y_2 \lambda x_1$ $\mathbf{R}xy$ | $\lambda z_{-1}\{z_2\}$ |
| | $J_1 \times J_{-1}$ | $\wedge\{ \lambda x_1 \mathbf{R}xy \times y_{-2} \mid \mathbf{W}y \}$ | | $\varnothing$ | $\lambda w_{-2}\{w_1\}$ | $\lambda z_{-1} \lambda x_1 \mathbf{R}xz$ | |
| $\lambda P \lambda Q$ $\{P \to Q\}$ | $\wedge\{ \mathbf{R}Jy \times J_{-1} \times y_{-2} \mid \mathbf{W}y \}$ | | | | | | |
| ※ | | | | $\lambda z_{-1} \lambda w_{-2} \mathbf{R}wz$ | | | |
| ※ | | | | | | | |

4. if Jay doesn't respect any woman, then she doesn't respect him

| if | Jay [+1, −1] | doesn't | respect | any woman [+2, −2] | then she doesn't respect him |
|---|---|---|---|---|---|
| | | | $\lambda y_2 \lambda x_1 \mathbf{R}xy$ | Л$\{ y_2 \times y_{-2} \mid \mathbf{W}y \}$ | |
| | | $\lambda P_1 \lambda x_1 \sim Px$ | Л$\{ \lambda x_1 \mathbf{R}xy \times y_{-2} \mid \mathbf{W}y \}$ | | |
| | $J_1 \times J_{-1}$ | $\wedge\{ \lambda x_1 \sim\mathbf{R}xy \times y_{-2} \mid \mathbf{W}y \}$ | | | |
| $\lambda P \lambda Q\{P \to Q\}$ | $\wedge\{ \sim\mathbf{R}Jy \times J_{-1} \times y_{-2} \mid \mathbf{W}y \}$ | | | | |
| ※ | | | | | (from below) $\lambda z_{-1}\ \lambda w_{-2} \sim\mathbf{R}wz$ |
| ※ | | | | | |

| then | (−2) she [+1] | doesn't | respect | (−1) him [+2] |
|---|---|---|---|---|
| | | | $\lambda y_2 \lambda x_1 \mathbf{R}xy$ | $\lambda z_{-1}\{z_2\}$ |
| | | $\lambda P_1 \lambda x_1 \sim Px$ | $\lambda z_{-1}\ \lambda x_1\ \mathbf{R}xz$ | |
| $\varnothing$ | $\lambda w_{-2}\{w_1\}$ | $\lambda z_{-1}\ \lambda x_1 \sim\mathbf{R}xz$ | | |
| $\lambda z_{-1}\ \lambda w_{-2} \sim\mathbf{R}wz$ | | | | |

# 21. Appendix 1 – The Logic of Junctions

## 1. Junctions[27]

| | | |
|---|---|---|
| $\wedge$ | conjunction | every |
| $\vee$ | disjunction | some |
| Л | subjunction | any |
| ⊠ | exclusive-disjunction | exactly one |
| $\Sigma$ | sum | a (base) |
| $\Pi$ | product | a (promoted) |

## 2. Types

the usual rules, plus:

if $\mathfrak{I}$ is a type, and Ж is a junction, then Ж$\mathfrak{I}$ is a type.

## 3. Type-Identities

| | |
|---|---|
| $\wedge$S = S | ЛS $\neq$ S |
| $\vee$S = S | |
| $\Sigma$S = S | $\Sigma$D = D |
| ⊠S = S | |

## 4. Formation Rules

if Ж is a junction, $\nu$ is a variable, $\varepsilon$ is an expression of type $\mathfrak{I}$, and $\Phi$ is a formula, then

Ж$_\nu\{\varepsilon \mid \Phi\}$

is an expression of type Ж$\mathfrak{I}$.

## 5. Informal Rule

if $\varepsilon$ and $\Phi$ share exactly one free variable, then that is understood to be the variable of abstraction, in which case we simply write the following informal expression.

Ж$\{\varepsilon \mid \Phi\}$

## 6. Simplification Principles

| | | | |
|---|---|---|---|
| $\wedge_\nu\{\Psi \mid \Phi\}$ | / | $\forall\nu\{\Phi \rightarrow \Psi\}$ | [$\wedge$-simplification] |
| $\vee_\nu\{\Psi \mid \Phi\}$ | / | $\exists\nu\{\Phi \;\&\; \Psi\}$ | [$\vee$-simplification] |
| $\Sigma_\nu\{\Psi \mid \Phi\}$ | / | $\exists\nu\{\Phi \;\&\; \Psi\}$ | [$\Sigma$-simplification] |
| ⊠$_\nu\{\Psi \mid \Phi\}$ | / | $\exists!\nu\{\Phi \;\&\; \Psi\}$[28] | [⊠-simplification] |
| $\Pi_\nu\{\Psi \mid \Phi\}$ | / | $\forall\nu\{\Phi \rightarrow \Psi\}$ | [$\Pi$-simplification] |
| provided the node is assertoric[29] | | | |

[27] We have only introduced $\wedge$, $\vee$, and Л in the current chapter. $\Sigma$ and $\Pi$ are introduced in the chapter "Indefinite Noun Phrases", and ⊠ is introduced in the chapter "Numerical Quantification".

[28] $\exists!\nu\Phi =_{df} \exists\omega\forall\nu\{\Phi \leftrightarrow \nu{=}\omega\}$, where $\omega{\neq}\nu$ and $\omega$ is not free in $\Phi$.

[29] For current purposes, a node is assertoric precisely if it is a (closed, complete) sentence *and* it is the top node.

## 7. Junction-Composition (Ж-Com)

| | |
|---|---|
| $\alpha$ | $\alpha$ is any expression, or null |
| $Ж_{\nu}\{\beta \mid \Phi\}$ | Ж admits $\alpha$ |
| $\{\alpha,\beta\} \vdash \gamma$ | a sub-derivation of $\gamma$ from $\{\alpha,\beta\}$ |
| $Ж^{*}_{\nu}\{\gamma \mid \Phi\}$ | Ж* is a transform of Ж |

The nature of Ж*, and the admissibility restrictions are as follows.

(1) Σ admits no junctions except Σ.

(2) ∧ does not admit
- (1) relative-pronoun-phrases.
- (2) anti-tonic functions, except
  - (1) ⟦every⟧
  - (2) ⟦every F of⟧
  - (3) ⟦if it is F⟧

(3) Л admits all functions;
anti-tonic functions transform (promote) Л to ∧.[30]

(4) Σ admits all functions;
anti-tonic functions transform (promote) Σ to Π.

## 8. Set-Abstracts and Generalized Set-Abstracts

The official formation-rule for junctions specifies that

$$Ж_{\nu}\{\varepsilon \mid \Phi\}$$

has type Ж$\mathfrak{I}$ when $\varepsilon$ has type $\mathfrak{I}$ and $\Phi$ is a formula. The variable $\nu$ formally specifies which variable in $\{\varepsilon \mid \Phi\}$ is bound. But, often, this is clear, so $\nu$ is omitted, in which case we obtain the following (informal) expression.

$$Ж\{\varepsilon \mid \Phi\}$$

For example, if $\varepsilon$ is an individual-variable $\nu$, we have the following.

$$Ж\{\nu \mid \Phi\}$$

Now, we take the sub-expression

$$\{\nu \mid \Phi\}$$

to be a set-abstract, which informally reads

the set of all $\nu$ such that $\Phi$

[30] Other functions and contexts, including modal, interrogative, and permitative, also promote Л and Σ.

which is governed by the following set-abstraction principle [where $\in$ is set-membership].

$$\forall\nu\{\nu \in \{\nu \mid \Phi\} \leftrightarrow \Phi\}$$

We also admit generalized set-abstracts, which is to say we admit set-abstracts of the form

$$\{\varepsilon \mid \Phi\}$$

where $\varepsilon$ is any expression and $\Phi$ is any formula – officially defined as follows.

$$\{\varepsilon \mid \Phi\} =_{df} \quad \{w \mid \exists\nu_1\ldots\exists\nu_k\,(\Phi \;\&\; \nu = \varepsilon)\}$$

here, $w$ is any variable not free in either $\varepsilon$ or $\Phi$, and $\nu_1, \ldots, \nu_k$ are all the variables that are free in both $\varepsilon$ and $\Phi$

For example, in arithmetic, the set described as follows

$$\{x^2 \mid x \text{ is even}\}$$
$$=_{df}$$
$$\{y \mid \exists x(x \text{ is even} \;\&\; y = x^2)\}$$

is the set of squares of even numbers. An example with two bound variables, the following is a fairly standard set-theoretic definition of the Cartesian product of two sets.

$$A \times B \quad =_{df} \quad \{\langle x,y\rangle \mid x{\in}A \;\&\; y{\in}B\}$$
$$=_{df} \quad \{z \mid \exists x\exists y(x{\in}A \;\&\; y{\in}B \;\&\; z{=}\langle x,y\rangle)\}$$

In other words, the Cartesian product of sets $A$ and $B$ is that set that consists of precisely those ordered pairs whose first component is in $A$ and whose second component is in $B$.

## 9. A Variable-Binding Confound

The usual (informal) convention in mathematics is that repeated variables are bound and non-repeated variables are free. Unfortunately, when applied to junctions, such as

$$\bigwedge\{\varepsilon \mid \Phi\}$$

this convention is a potential source of chaos – specifically, when $\varepsilon$ and $\Phi$ share two or more free variables. In those cases, we *must* employ the official expression

$$\bigwedge_\nu\{\varepsilon \mid \Phi\}$$

to specify which variable is bound. In this case, the (odd-looking) sub-expression

$${}_\nu\{\varepsilon \mid \Phi\}$$

may be understood as follows.

$${}_\nu\{\varepsilon \mid \Phi\} \quad =_{df} \quad \{w \mid \exists\nu(\Phi \;\&\; w = \varepsilon)\}$$

Here, $w$ is any variable not free in $\varepsilon$ or $\Phi$.

## 22. Appendix 2 – Monotonic and Anti-Tonic Functions

In mathematics, a function $\phi$ is said to be *monotonic-increasing* precisely if:

for any $x,y$, if $x \leqslant y$, then $\phi(x) \leqslant \phi(y)$

where $\leqslant$ is any **order-relation**, which is to say any relation that is transitive and reflexive. Also, $\phi$ is said to be *monotonic-decreasing* precisely if:

for any $x,y$, if $x \leqslant y$, then $\phi(y) \leqslant \phi(x)$

Often, the terminology is abbreviated, and the former is simply called 'monotonic', and the latter is simply called 'anti-tonic'. We adopt the abbreviated terminology.

The obvious question is: how does '$\leqslant$' apply to type-theoretic semantics? The following is the proposed definition, which applies inductively to all (non-junctional) types.

| | | | | |
|---|---|---|---|---|
| (a1) | $S_1 \leqslant S_2$ | $=_{df}$ | $S_1 \rightarrow S_2$ | if $S_1$ and $S_2$ are sentences (S) |
| (a2) | $n_1 \leqslant n_2$ | $=_{df}$ | $\forall P\{P[n_1] \rightarrow P[n_2]\}$[31] | if $n_1$ and $n_2$ are names (D) |
| (a3) | $\phi_1 \leqslant \phi_2$ | $=_{df}$ | $\forall \alpha\{\phi_1(\alpha) \leqslant \phi_2(\alpha)\}$ | where $\phi_1$ and $\phi_2$ are functors of the same type, and $\alpha$ is an argument-variable of the appropriate type |

## 1. Examples of Anti-Tonic Functions

### 1. not

We take ⟦not⟧ to be the usual truth-function $\lambda\Phi{\sim}\Phi$. To show that this is anti-tonic, we first note that it amounts to the following.

$\forall P\forall Q\{ P \leqslant Q \rightarrow {\sim}Q \leqslant {\sim}P \}$

Given the definition of $\leqslant$, this amounts to

$\forall P\forall Q\{ P \rightarrow Q \;.\!\rightarrow\!.\; {\sim}Q \rightarrow {\sim}P \}$

which is simply a quantified version of the law of contraposition.

### 2. if

We take ⟦if⟧ to be the usual truth-function $\lambda\Phi\lambda\Psi\{\Phi\rightarrow\Psi\}$, which we abbreviate $\lambda\Phi\{\text{if}(\Phi)\}$. To show that this is anti-tonic, we first note that it amounts to the following.

$\forall P\forall Q\{ P \leqslant Q \rightarrow \text{if}(Q) \leqslant \text{if}(P) \}$

Here, P and Q are sentences. Given the definition of $\leqslant$, this amounts to:

$\forall P\forall Q\{ P \rightarrow Q \;.\!\rightarrow \forall R(\text{if}(Q)(R) \rightarrow \text{if}(P)(R) \}$

[31] The domain over which 'P' ranges must be limited to "proper" predicates, the main purpose of which is to rule out predicates like '$\lambda x[x=a]$' [which denotes the property of being identical to $a$]. If we allow the latter predicate, then we have $a \leqslant b$ iff $a=b$, which has the unfortunate consequence that every one-place predicate is both monotonic and anti-tonic.

which given the definition of 'if(Φ)' amounts to:

$$\forall P\forall Q\{\ P{\rightarrow}Q\ .{\rightarrow}\ \forall R(Q{\rightarrow}R\ .{\rightarrow}.\ P{\rightarrow}R)\ \}$$

which is simply the second-order version of the law of transitivity for →.

## 3. every

For these purposes, we take ⟦every⟧ to be $\lambda P\lambda Q\forall x\{Px{\rightarrow}Qx\}$, which we abbreviate $\lambda P\{\text{every}(P)\}$. To show that this is anti-tonic, we first note that this amounts to the following.

$$\forall P\forall Q\{\ P \leqslant Q\ \rightarrow\ \text{every}(Q) \leqslant \text{every}(P)\ \}$$

Here, P and Q are one-place predicates. So, given the definition of ⩽, this amounts to:

$$\forall P\forall Q\{\ \forall x\{Px{\rightarrow}Qx\}\ .{\rightarrow}\ \forall R(\text{every}(Q)(R) \rightarrow \text{every}(P)(R)\ \}$$

which amounts to:

$$\forall P\forall Q\{\ \forall x\{Px{\rightarrow}Qx\}\ .{\rightarrow}\ \forall R(\forall x\{Qx{\rightarrow}Rx\}\ .{\rightarrow}.\ \forall x\{Px{\rightarrow}Rx\})\ \}$$

which is simply the second-order version of the law of transitivity for ∀.

## 4. no

⟦no⟧ works pretty much just like ⟦every⟧; proving it is anti-tonic is left as an exercise.

## 5. no A

Whereas both ⟦every⟧ and ⟦no⟧ are anti-tonic, ⟦every A⟧ is monotonic, but ⟦no A⟧ is anti-tonic. For these purposes, we take ⟦no A⟧ to be $\lambda P\forall x\{Ax{\rightarrow}{\sim}Px\}$ [which we abbreviate $\lambda P\{\text{no}(A)(P)\}$. To show that this is anti-tonic, we first note that this amounts to the following.

$$\forall P\forall Q\{\ P \leqslant Q\ \rightarrow\ \text{no}(A)(Q) \leqslant \text{no}(A)(P)\ \}$$

Here, P and Q are one-place predicates. So, given the definition of ⩽, this amounts to:

$$\forall P\forall Q\{\ \forall x\{Px{\rightarrow}Qx\}\ .{\rightarrow}.\ \forall x\{Ax{\rightarrow}{\sim}Qx\} \rightarrow \forall x\{Ax{\rightarrow}{\sim}Px\}\ \}$$

which is a second-order version of a quantification principle [if every P is Q, then if no A is Q, then no A is P].

## 6. every F of… [genitive 'of']

For these purposes, we take ⟦every F of⟧ to be $\lambda y\lambda P\forall x\{Fxy{\rightarrow}Px\}$. To show this is anti-tonic, we first note that this amounts to the following.

$$\forall x\forall y\{\ x \leqslant y\ \rightarrow\ \lambda P\forall z\{Fxz{\rightarrow}Px\} \leqslant \lambda P\forall z\{Fyz{\rightarrow}Py\}$$

So, given the definition of ⩽, this amounts to:

$$\forall x\forall y\{\ \forall P\{Px{\rightarrow}Py\}\ \rightarrow\ \forall P\{\forall z\{Fxz{\rightarrow}Px\} \rightarrow \forall z\{Fyz{\rightarrow}Py\}\}$$

the proof of which goes as follows.

| | | |
|---|---|---|
| (1) | ~~SHOW~~: $\forall x \forall y\{\ \forall P\{Px \rightarrow Py\}\ \rightarrow\ \forall P\{\forall z\{Fyz \rightarrow Py\} \rightarrow \forall z\{Fxz \rightarrow Px\}\}$ | U2CD |
| (2) | $\forall P\{Px \rightarrow Py\}$ | AS |
| (3) | ~~SHOW~~: $\forall P\{\forall z\{Fbz \rightarrow Pb\} \rightarrow \forall z\{Fyz \rightarrow Pa\}\}$ | UCD |
| (4) | $\forall z\{Fbz \rightarrow Pb\}$ | As |
| (5) | ~~SHOW~~: $\forall z\{Faz \rightarrow Pa\}$ | UCD |
| (6) | $Fac$ | As |
| (7) | ~~SHOW~~: $Pb$ | DD |
| (8) | $[\lambda x Fxc]a$ | 6,$\lambda$C |
| (9) | $[\lambda x Fxc]b$ | 2,8,$\forall\rightarrow$O |
| (10) | $Fbc$ | 9,$\lambda$C |
| (11) | $Pb$ | 4,10,$\forall\rightarrow$O |

## 7. λx(if x is F)

This corresponds to the sentence ‘if it is F’ where ‘it’ is essentially-anaphoric. To show that this is anti-tonic, we first note that this amounts to the following.

$$\forall x \forall y\{\ x \leqslant y\ \rightarrow\ \mathrm{if}(Fy) \leqslant \mathrm{if}(Fx)\ \}$$

Here, $x$ and $y$ are entities. So, given the definition of $\leqslant$, this amounts to:

$$\forall x \forall y\{\ \forall P\{Px \rightarrow Py\} \rightarrow \forall Q\{\mathrm{if}(Fy)(Q) \leqslant \mathrm{if}(Fx)(Q)\ \}$$

which given the definition of ⟦if⟧ and $\leqslant$ amounts to:

$$\forall x \forall y\{\ \forall P\{Px \rightarrow Py\} \rightarrow \forall Q\{\ (Fy \rightarrow Q) \rightarrow (Fx \rightarrow Q)\ \}\ \}$$

the proof of which goes as follows.

| | | |
|---|---|---|
| (1) | ~~SHOW~~: $\forall x \forall y\{\ \forall P\{Px \rightarrow Py\} \rightarrow \forall Q\{\ (Fy \rightarrow Q) \rightarrow (Fx \rightarrow Q)\ \}\ \}$ | U2CD |
| (2) | $\forall P\{Pa \rightarrow Pb\}$ | As |
| (3) | ~~SHOW~~: $\forall Q\{\ (Fb \rightarrow Q) \rightarrow (Fa \rightarrow Q)\ \}$ | UCD |
| (4) | $Fb \rightarrow Q$ | As |
| (5) | ~~SHOW~~: $Fa \rightarrow Q$ | CD |
| (6) | $Fa$ | As |
| (7) | ~~SHOW~~: $Q$ | DD |
| (8) | $Fb$ | 2,6,$\forall\rightarrow$O |
| (9) | $Q$ | 4,8,$\rightarrow$O |