1. A Few More Applications of Expanded Categorial Semantics

1. Generalized-Conjunction

So far, we treat the word ‘and’ as a two-place connective, with the following type.

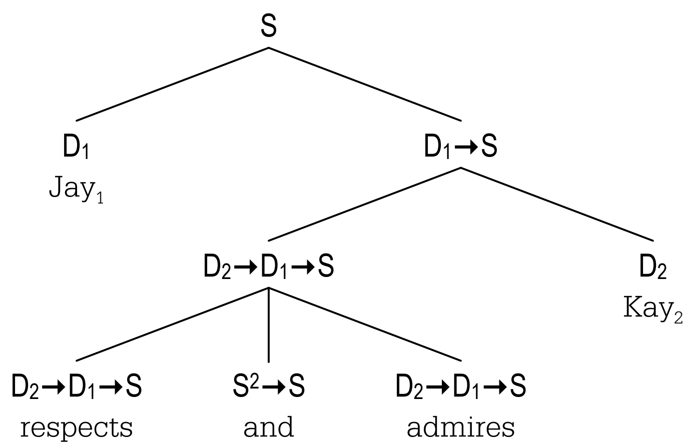
type[and]	=	$S^2 \rightarrow S$
	$=_{df}$	$(S \times S) \rightarrow S$
	[≡	$S \rightarrow (S \rightarrow S)$]

If we only have simple-composition [function-application] available, then we can use ‘and’ to combine two sentences to form another sentence, but we cannot combine other types of expressions. On the other hand, if we avail ourselves of expanded-composition, underwritten by categorial logic, then we can combine many other types of expressions.[1] What's more, we get pretty much exactly what we expect semantically.

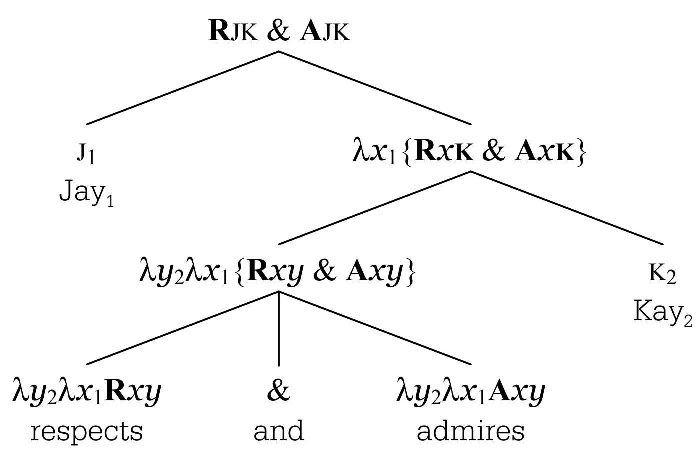
1. Example 1

Jay respects and admires Kay

In this example ‘and’ combines two transitive verbs, and the natural parsing looks thus.



And the following is the corresponding semantic-tree.



Note 1: Henceforth, when we write ‘&’ by itself like this, we mean it to be short for ‘λPQ{P&Q}’.

Note 2: Henceforth, we appeal to the idea that atomic sentences in logic are zero-place predicates, and accordingly use upper-case Roman letters as sentence-variables. Officially, the predicate-letters should be marked as to degree – e.g. P^0, P^1, P^2 – but generally this can be kept straight contextually. This allows us to place the Greek letters back in the meta-language.

[1] There is a certain amount of over-generation, which means the syntactic module has to intervene. See later example. The basic idea is that ‘and’ is a coordinator, which means it combines items of the same conventional syntactic type [which may or may not agree with the categorial type].

The salient question is how do we combine ⟦respects⟧, ⟦and⟧, and ⟦admires⟧. This is accomplished by way of the following derivation.

respects + and + admires

(1)	$\lambda y_2 \lambda x_1 \mathbf{R}xy$	D₂→D₁→S	1	Pr
(2)	$\lambda y_2 \lambda x_1 \mathbf{A}xy$	D₂→D₁→S	2	Pr
(3)	$\lambda PQ\{P\&Q\}$	S×S.→S	3	Pr
(4)	~~SHOW~~: $\lambda y_2 \lambda x_1 \{\mathbf{R}xy \ \& \ \mathbf{A}xy\}$	D₂→D₁→S	123	λλD
(5)	y_2	D₂	4	As
(6)	x_1	D₁	5	As
(7)	~~SHOW~~: $\mathbf{R}xy \ \& \ \mathbf{A}xy$	S	12345	DD
(8)	$\lambda x_1 \mathbf{R}xy$	D₁→S	14	1,5,λO
(9)	$\mathbf{R}xy$	S	145	6,8,λO
(10)	$\lambda x_1 \mathbf{A}xy$	D₁→S	24	2,5,λO
(11)	$\mathbf{A}xy$	S	245	6,10,λO
(12)	$\mathbf{R}xy \times \mathbf{A}xy$	S×S	1245	9,11,×I
(13)	$\mathbf{R}xy \ \& \ \mathbf{A}xy$	S	12345	3,12,λO

As it turns out, the above derivation is a special case of a considerably more general schema, which we call **generalized-conjunction**, which is given as follows.

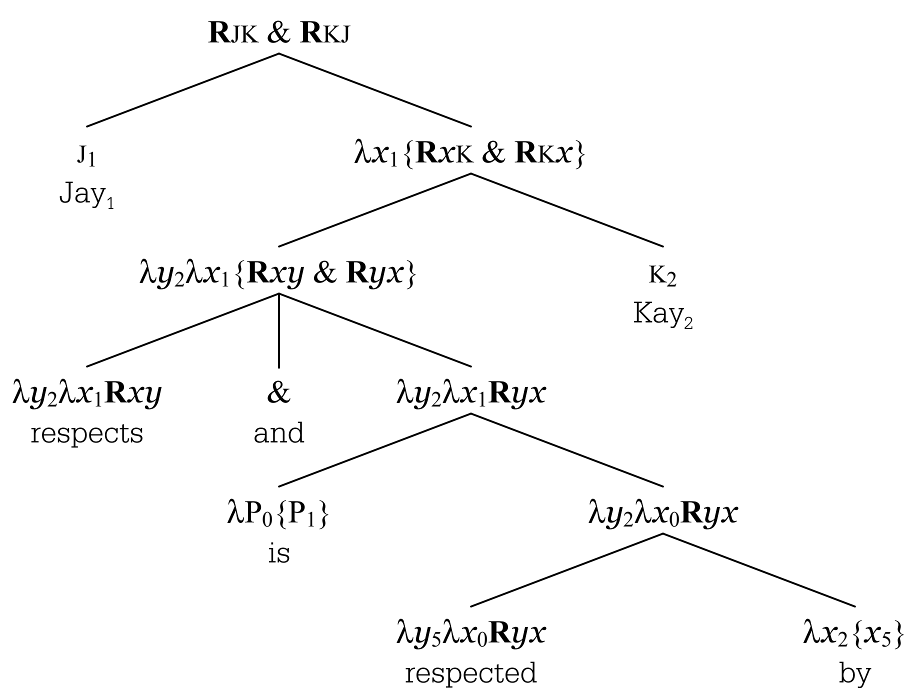
$$\lambda \nu^1 \ldots \lambda \nu^k \Phi \ ; \ \lambda \nu^1 \ldots \lambda \nu^k \Psi \ ; \ \lambda PQ\{P\&Q\} \ \vdash \ \lambda \nu^1 \ldots \lambda \nu^k \{\Phi \& \Psi\}$$

Here, Φ and Ψ are formulas, and $\nu^1 \ldots \nu^k$ are distinct variables of any types, possibly with case-markers.

2. Example 2

Jay respects, and is respected by, Kay

Here, the word 'and' combines a transitive verb with a quasi-transitive-verb.[2] The following is the computation.

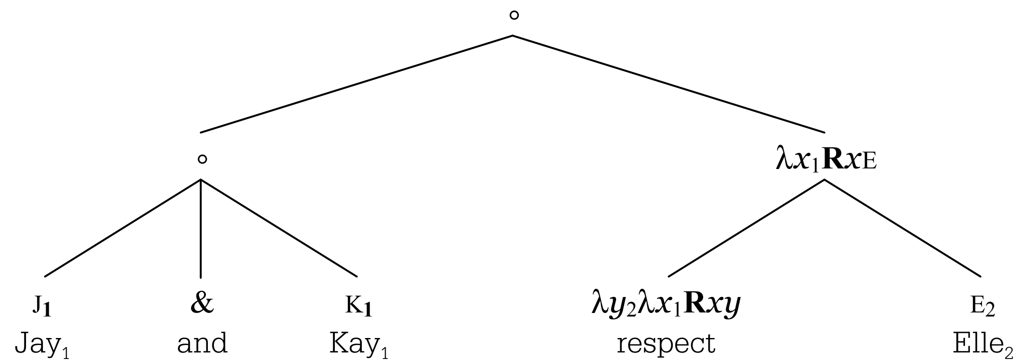


[2] The construction of 'is respected by' combines 'respected' with 'by', which is a case-marker, which means we have to relax our syntactic requirement about case-markers attaching *only* to NPs. Also, note that 'by' has type D₂→D₅, which allows us to render 'is respected by' as having type D₂→D₁→S [i.e., transitive verb].

3. Example 3

Jay and Kay respect Elle

In this example, 'and' combines two D's. The following is a first approximation to the semantic-tree.

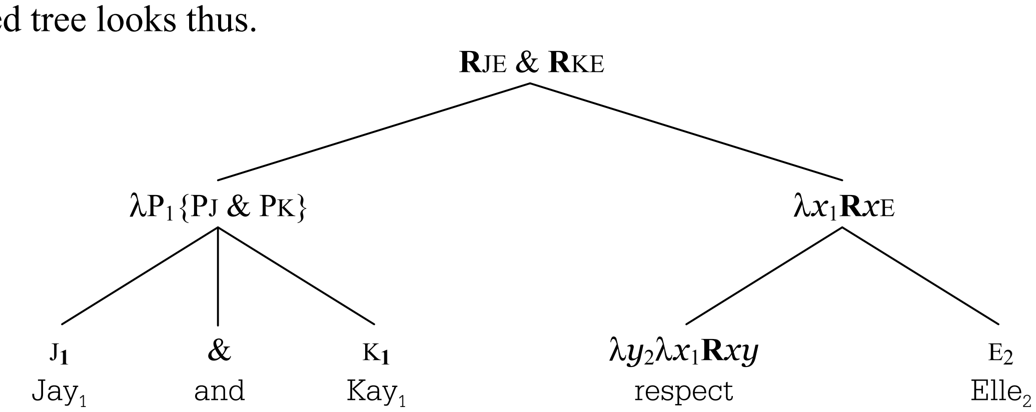


The next question is how 'and' combines categorially with two (inflected) D's. Once again, although this is impossible by simple-composition, it is possible by expanded-composition, as seen in the following derivation. [For the sake of clutter-reduction, we omit the case-markers.]

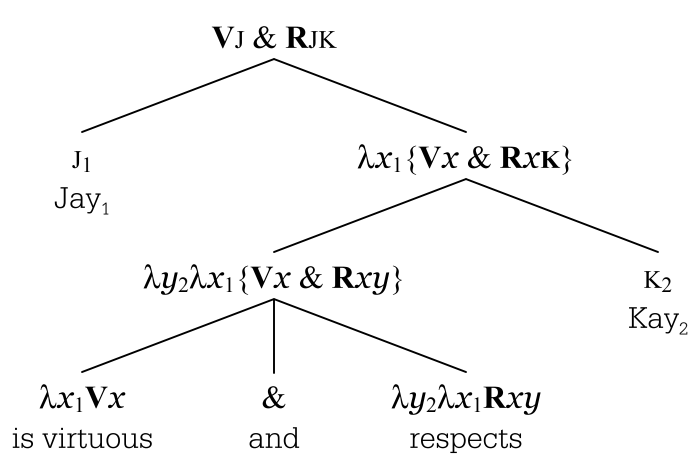
Jay + and + Kay

(1)	J	D	1	Pr
(2)	K	D	2	Pr
(3)	λPQ{P&Q}	S×S.→S	3	Pr
(4)	~~SHOW~~: λP{PJ & PK}	D→S.→S	123	λD
(5)	│ P	D→S	4	As
(6)	│ ~~SHOW~~: PJ & PK	S	1234	DD
(7)	│ │ PJ	S	14	1,5,λO
(8)	│ │ PK	S	24	2,5,λO
(9)	│ │ PJ × PK	S×S	124	7,8,×I
(10)	│ │ PJ & PK	S	1234	3,9,λO

The completed tree looks thus.



4. Example 4 [Syntactic-Rule Required to Avoid Odd Conjunction]



2. Ambiguity

According to the received view, ambiguity comes in two varieties, known as *lexical-ambiguity* and *structural-ambiguity*.

1. Lexical-Ambiguity

Lexical-ambiguity occurs when the same surface-expression (pronunciation) conveys two or more distinct morphemes – alternatively, when a single morpheme has two or more distinct entries in the lexicon.

A prominent logical example is ‘and’ which has two[3] different meanings, as in:

> Jay respects Kay **and** Elle
> Jay is between Kay **and** Elle

Another example is ‘ed’, which has at least three inflectional uses in English:

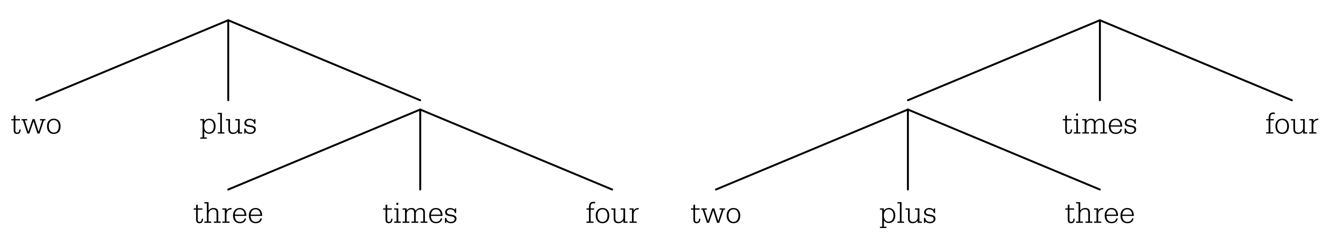
Jay is respect**ed** by Kay	[passive voice]
Kay mov**ed** here last year	[past tense (+perfective aspect)]
Jay has liv**ed** here for two years	[perfect aspect]

2. Structural (Scope) Ambiguity

When three or more phrases are combined, the *order* of combination may affect the meaning of the resulting phrase. For example, consider the following two phrases.

> two plus three times four
> baby whale kill er

More technically speaking, structural-ambiguity occurs when two different trees give rise to the same surface expression (string of morphemes). For example, the first phrase above admits two different tree-structures, as follows.



Structural-ambiguity includes *scope-ambiguity*; for example, the semantic question in the first one is whether ‘plus’ or ‘times’ has wide-scope.

3. Multiple-Ambiguity

Some phrases are multiply-ambiguous.

1. poor violin play er
2. buffalo buffalo buffalo buffalo …
3. the horse walked by the trainer [walked the fastest]
4. come see the man eating chicken and fish
5. show dogs don't tailgate

For example, in the first one, ‘poor’ has three relevant lexical entries [poor *versus* rich, poor *versus* good, poor(ly) *versus* well]. It is also structurally ambiguous according to whether the player is poor or the violin in poor.

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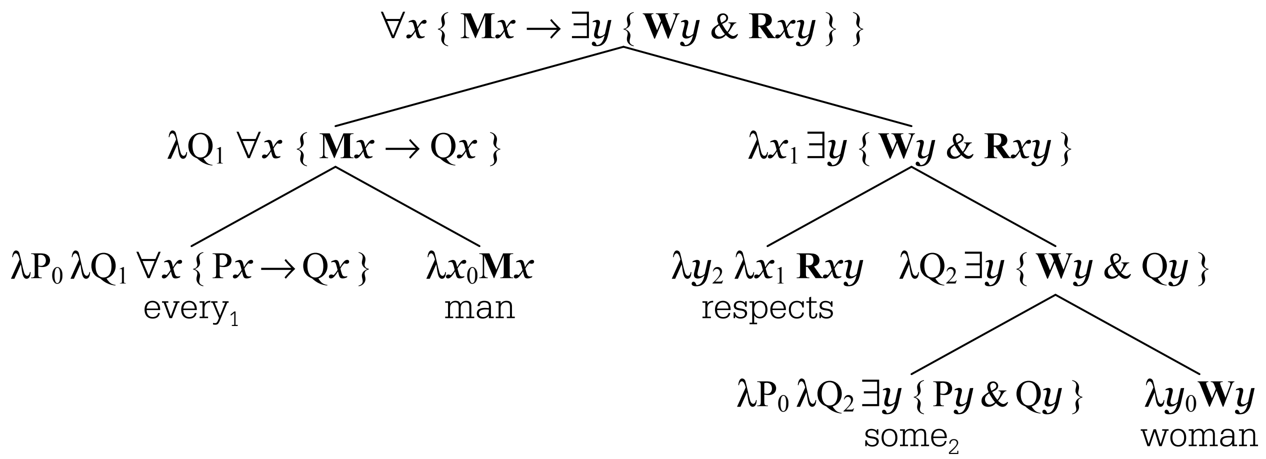
[3] It actually has other meanings as well, which we discuss later.

3. A Key Example

The following example from elementary logic is a key example of a sentence that involves structural/scope ambiguity.

every man respects some woman

This plausibly has a standard SVO form, which is semantically analyzed as follows (where, as usual, the case-markers are "internalized").[4]



However, in addition to this reading according to which 'every man' has wide-scope, there is another (less obvious) reading according to which 'some woman' has wide-scope, in which case the following is its translation.

$$\exists y \{ \mathbf{W}y \,\&\, \forall x \{\mathbf{M}x \to \mathbf{R}xy\} \}$$

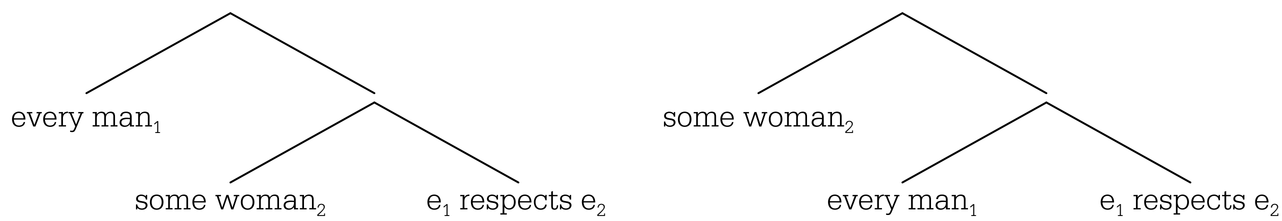
In this connection, notice the following series.

every man respects some woman
some woman is respected by every man
there is some woman who is respected by every man

There are three accounts of this ambiguity.

1. Logical-Form Ambiguity

According to this account, which is the more or less standard "logical" account of quantifier-scope, in order to obtain the logical-form of the above sentence, both quantifier phrases are raised to their logical position, which is at the front of their containing clause. Since there are two landing positions for the quantifier phrases, there are correspondingly two logical-forms, given roughly as follows.



Here, e_1 is the pronoun-*trace* of 'every man' and e_2 is the pronoun-*trace* of 'some woman'.[5] One must further postulate that the two QPs are lowered back into their respective trace positions in the final phonetic-form.

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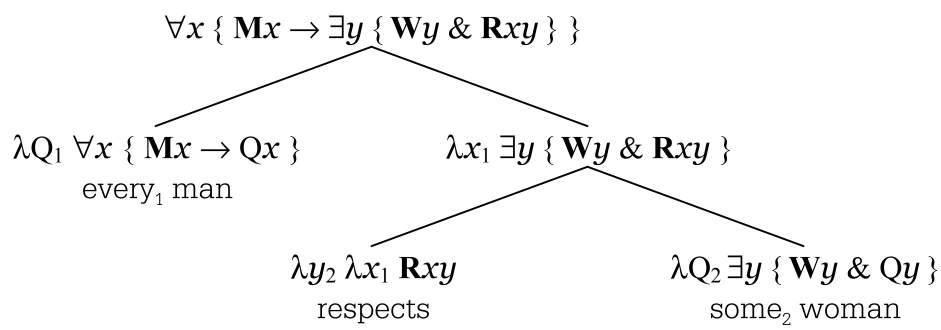
[4] Note that, in the tree, we "pre-infect" the quantifiers. Although this is categorially perfectly legal, it is not *syntactically* permitted, since quantifiers are not NPs. But since quantifiers ultimately reside inside NPs, which can be case-marked, we do this simply to save space in the tree.

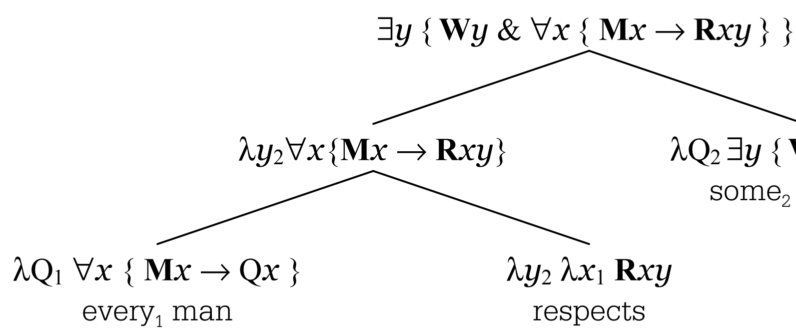
[5] Traces behave *a lot* like pronouns, which is how one does scope in elementary logic.

2. Structural-Ambiguity

According to this account, the relative scopes of 'every man' and 'some woman' are indicated in the underlying tree-structure. In particular, in addition to the tree-structure given earlier,



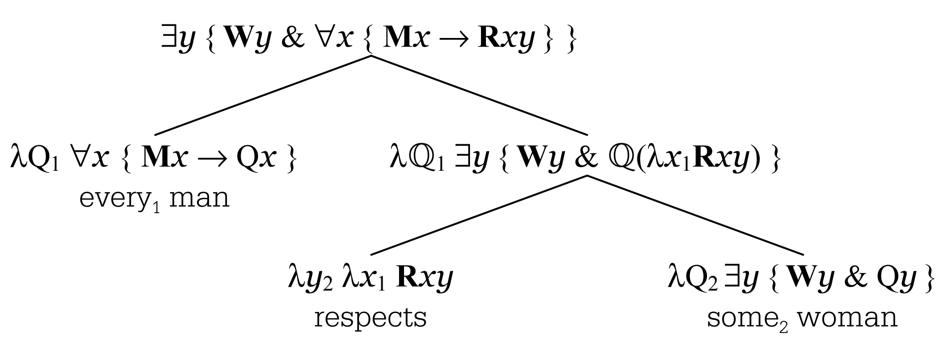
the following structure is also admissible.



According to this analysis, we first combine 'every$_1$ man' and 'respects', which results in an accusative-predicate [$D_2 \rightarrow S$], and then combine the resulting expression with 'some$_2$ woman', which takes an accusative-predicate and delivers a sentence.

3. Compositional-Ambiguity

What is perhaps surprising is that one can achieve the same semantic result without adjusting the tree, but rather noticing that one of the nodes admits an alternative computation, as seen in the following.



The following is the key derivation, which is an application of the general inference-principle we call "inflection".

(1)	$\lambda y_2\, \lambda x_1\, \mathbf{R}xy$	$D_2 \rightarrow D_1 \rightarrow S$	1	Pr
(2)	$\lambda Q_2\, \exists y\,\{\,\mathbf{W}y\;\&\;Qy\,\}$	$D_2 \rightarrow S. \rightarrow S$	2	Pr
(3)	$\mathbb{Q}_1$ $[=_{df}\, \lambda P_1 \mathbb{Q}(P)]$	$D_1 \rightarrow S. \rightarrow S$	3	As
(4)	$\lambda y_2\, \mathbb{Q}_1(\lambda x_1\, \mathbf{R}xy)$	$D_2 \rightarrow S$	13	1,4,Trans
	$\lambda y_2\, \mathbb{Q}([\lambda x \mathbf{R}xy])$			λC
(5)	$[\lambda Q_2\, \exists y\,\{\,\mathbf{W}y\;\&\;Qy\,\}]\langle\, \lambda y_2\, \mathbb{Q}([\lambda x \mathbf{R}xy])\,\rangle$	S	123	2,4,λO
	$\exists y\,\{\,\mathbf{W}y\;\&\;[\lambda y \mathbb{Q}([\lambda x \mathbf{R}xy]\langle y\rangle\,\}$			λC
	$\exists y\,\{\mathbf{W}y\;\&\;\mathbb{Q}(\lambda x \mathbf{R}xy)\,\}$			λC
(6)	$\lambda \mathbb{Q}_1\, \exists y\,\{\,\mathbf{W}y\;\&\;\mathbb{Q}(\lambda x \mathbf{R}xy)\,\}$	$D_1 \rightarrow S. \rightarrow S. \rightarrow S$	12	3,5,λI

4. A More Complex Example

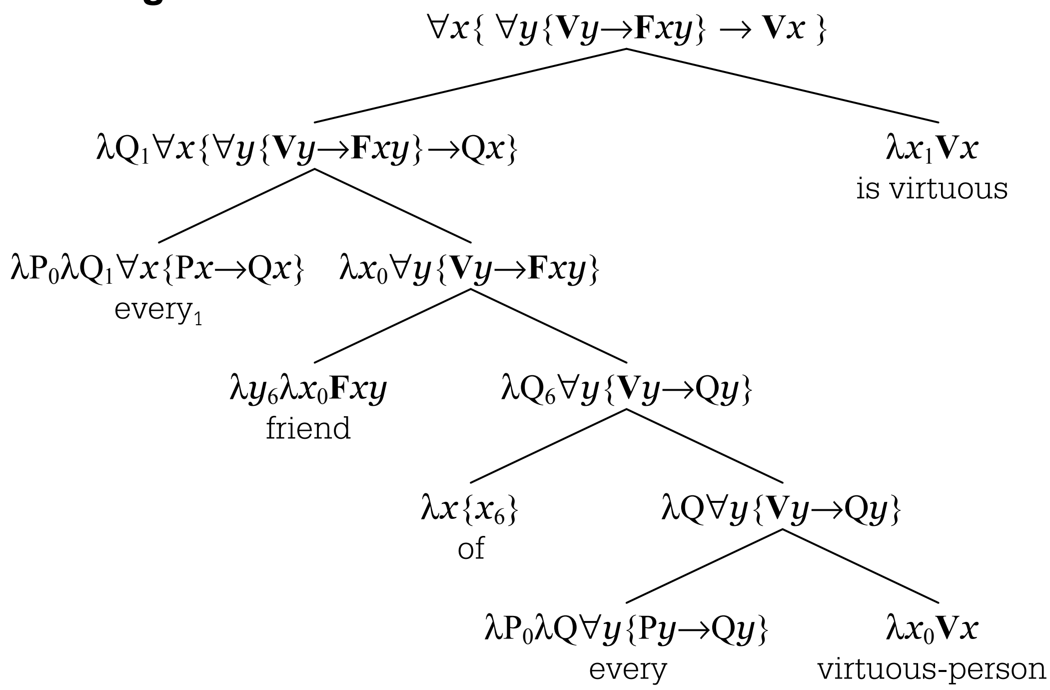
The following sentence is yet another example of quantifier scope-ambiguity.

every friend of every virtuous person is virtuous

The word ‘friend’ is an example of what we call a *genitive noun*, the type of which is very similar to the type of transitive verbs – the difference being the case-markings.

type(friend)	=	$D_6 \rightarrow (D_0 \rightarrow S)$	
⟦friend⟧	=	$\lambda y_6 \lambda x_0 \mathbf{F}xy$	[$\mathbf{F}[\alpha,\beta] =_{df}$ α is a friend of β]

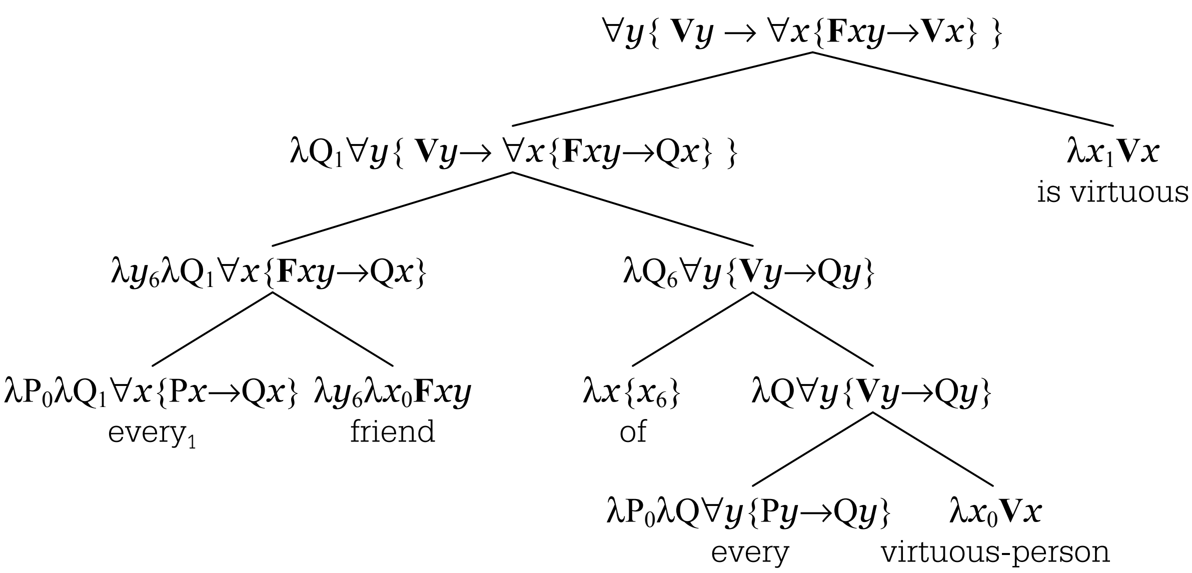
1. One Reading



This is not the most plausible reading of the sentence. Indeed, if we change the word ‘friend’ to ‘child’ or ‘wife’, then its plausibility drops significantly, for then its says that if you are a child/wife of every virtuous person, which is highly unlikely, then you are virtuous.

2. A More Plausible Reading

A more plausible reading reverses the respective scopes of the two quantifier phrases. Using the structural account of scope, we have the following alternative tree.



5. Combining Genitive Nouns with 'the'

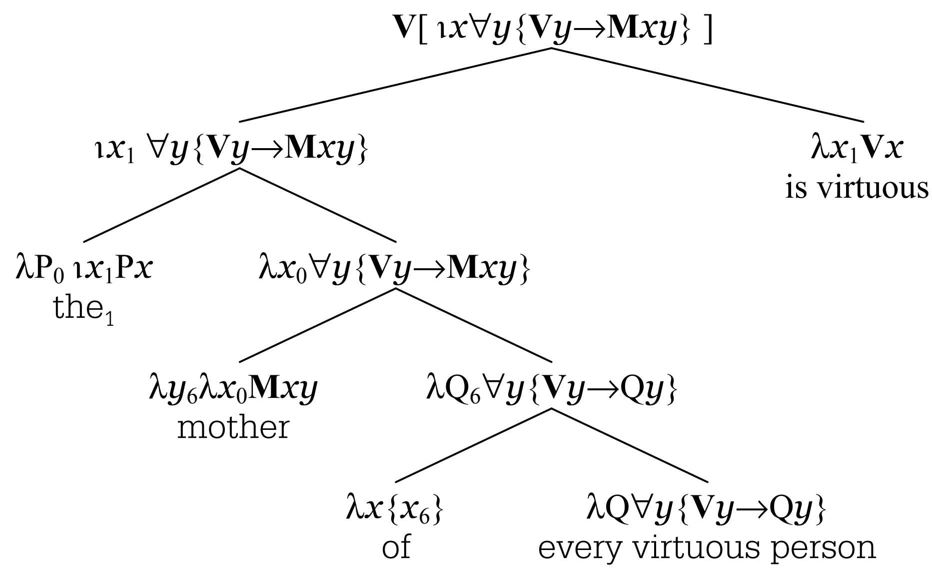
As noted earlier, many genitive-nouns – including 'mother' and 'father' – are *often* inherently definite, in which case they *in effect* have type $D_6 \rightarrow D$. This is not always the case, however, since many such words can be prefixed by 'the'. For this reason, our official line is that genitive-nouns all fundamentally have type $D_6 \rightarrow (D_0 \rightarrow S)$, but some occurrences carry with them a covert morpheme [def], which is equivalent to 'the'.[6]

The following is an example involving an overt definite-determiner.

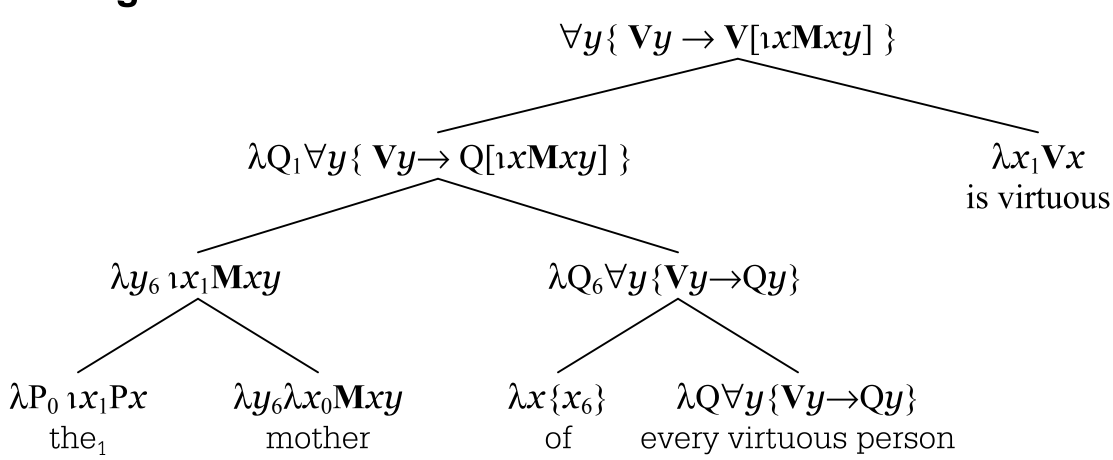
the mother of every virtuous-person is virtuous

Notice that the above sentence is very similar to our previous example; it is similarly ambiguous.

1. Reading 1



2. Reading 2



Note that we treat persons as the domain. Also note that we "pre-inflect" 'the' (see footnote 4). Finally, note that we allow the iota-operator to bind a complex expression 'x_1', its intuitive meaning being fairly clear ["the 1-marked entity x_1 such that…"].

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[6] This can also be understood as one (primary, indefinite) reading giving rise to another (secondary, definite) reading, in accordance with the following general principle, where G is a genitive noun and G^{δ} is its secondary (definite) reading.

$$G^{\delta} \quad =_{df} \quad \lambda x_6 \iota y G(x)(y_0)$$

The actual spelling of G^{δ} is a matter of practice. For example, in the case of 'mother', we could *in effect* have separate lexical entries '**M**' [mother] and 'ᴍ' [mother(def)] , related as follows.

$$\mathbf{M}[\alpha,\beta] \quad =: \quad \alpha \text{ is a mother of } \beta$$

$$\textsc{m}(\alpha) \quad =_{df} \quad \iota\nu\mathbf{M}[\nu,\alpha] \qquad [\nu \text{ not free in } \alpha]$$