1. Introduction

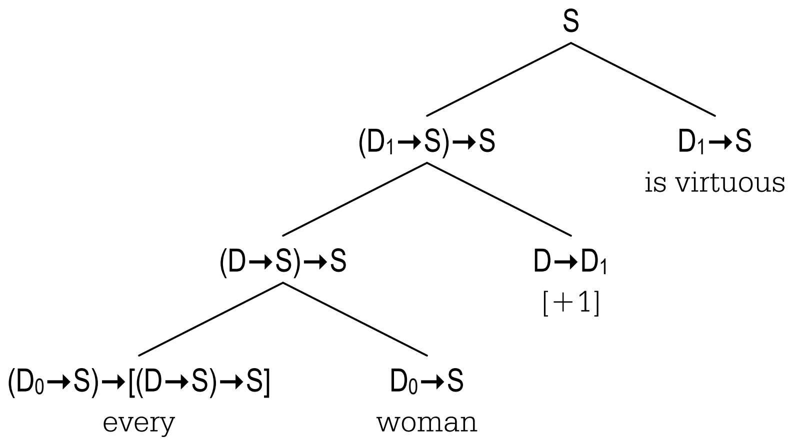
We have a theory of quantification and pronoun-binding, but it faces numerous difficulties – both methodological and empirical. In order to solve these difficulties, we propose a new account of quantifiers.

2. Our Old Theory of Quantifiers

According to our current theory, which traces to Montague, a quantifier is a two-place second-order predicate, which in particular has the following type.[1]

$$(D_0 \to S) \to [(D \to S) \to S]$$

In other words, a quantifier takes a common-noun phrase (0-inflected predicate), and delivers a function that takes an uninflected predicate, and delivers a sentence. The following syntactic-tree illustrates.



The intermediate phrase ‘every woman’ is what we call a ***quantifier-phrase***, which is a species of NP, which is an über-category. Basically, an NP is the sort of phrase that can be case-marked, and can be cross-referenced by a pronoun. For example, in the above tree ‘every woman’ is marked nominative [+1]. On the other hand, the phrase ‘is virtuous’ – whose further syntactic structure is ignored above – is a verb-phrase (VP), which is a one-place predicate that sub-categorizes for a single nominative argument. Also note that, in keeping with the second-order nature of quantifiers and quantifier-phrases, the QP ‘every woman’ takes the VP ‘is virtuous’ as an argument.

3. Our New Theory of Quantifiers

In what follows, we offer an alternative account of quantifiers in terms of a new class of type-theoretic objects, which we call ***junctions***, which are infinitary-operators.[2] Before we are finished, we will have proposed six different junctions. For the moment, we start with the simplest one, which is infinitary-conjunction.

[1] Here, D is the type of definite-noun-phrases (a.k.a. DNPs), and S is the type of (closed) sentences; alternatively, D is the type of entities (underlying elements of the domain), and S is the type of statements. We reserve the type E for events, which are employed to interpret eventive-verbs. [So far, our verbs are all stative.]
[2] By an infinitary-operator, we mean an operator that acts on *a collection of arguments* of arbitrary size, even infinite, and even empty.

4. Conjunction

First, we posit a new type-forming operator $\wedge$ – called 'conjunction',[3] defined so that

if $\mathfrak{I}$ is a type, then so is $\wedge\mathfrak{I}$.

This means that the following are examples of types.

$\wedge$D	conjunctions of D's
$\wedge$S	conjunctions of S's
$\wedge$(D→S)	conjunctions of predicates
$\wedge$(S→S)	conjunctions of connectives
$\wedge$D → S	functions from conjunctions of D's to S's
$\wedge$(D→S) → D	functions from conjunctions of predicates to D's

As with type-theory in general, although there is a staggering array of types, only a tiny fraction are in fact instantiated by any particular natural language.

We further propose the following type-identity

$\wedge$S = S

which reflects the intuition that the conjunction of any number of sentences is itself a sentence.

There is a corresponding expansion of the syntax of type-theory, which posits as well-formed all expressions of the following form,

$\wedge_\nu\{\varepsilon \mid \Phi\}$

which we propose to read as:

the conjunction (over ν) of all ε such that Φ

Here, ν is any variable, ε is any expression of type $\mathfrak{I}$, Φ is any formula, and the resulting expression has type $\wedge\mathfrak{I}$. Note that the variable ν officially identifies which variable is bound. On the other hand, in most applications, exactly one variable is free in both ε and Φ, in which case it is clear which variable is bound. In these circumstances, we omit ν, in which case we have (informal) expressions of the following form,

$\wedge\{\varepsilon \mid \Phi\}$

which we read:

the conjunction of all ε such that Φ.

[3] It is also called 'meet', which is borrowed from lattice theory, which borrows it (in effect) from projective geometry. Lattice-theoretic meet (a.k.a. *greatest lower bound*) is a generalization of logical-conjunction and set-intersection, and also "where two subspaces meet".

5. Quantifiers and Quantifier-Phrases

Our new type-logical apparatus allows us to reformalize universal quantifiers and quantifier-phrases. First, we read

$$\bigwedge\{x \mid \Phi\}$$

as

the conjunction of all x such that Φ.

So, for example, we can read

$$\bigwedge\{x \mid x \text{ is a man}\}$$

as

the conjunction of all x such that x is a man[4]

or more succinctly:

the conjunction of all men.

Intuitively, this amounts to

man_1 and man_2 and … and man_k

where we have enumerated the class of all men.

With this in mind, we offer our new proposal.

$$[\![\text{every}]\!] \quad = \quad \lambda P_0 \bigwedge\{x \mid Px\}$$

For example:

$$\begin{aligned} [\![\text{every man}]\!] &= \bigwedge\{x \mid x \text{ is a man}\} \\ &= \text{the conjunction of all men} \\ &\approx \text{man}_1 \text{ and } \text{man}_2 \text{ and } \ldots \text{ and } \text{man}_k \end{aligned}$$

6. Examples

1. Example 1

every man is virtuous

<table>
<tr><td></td><td>every</td><td>man</td><td>[+1]</td><td>is</td><td>virtuous</td><td></td></tr>
<tr><td></td><td>$\lambda P_0 \bigwedge\{x \mid Px\}$</td><td>$\mathbf{M}_0$</td><td></td><td>$\lambda P_0\{P_1\}$</td><td>$\mathbf{V}_0$</td><td></td></tr>
<tr><td>①</td><td colspan="2">$\bigwedge\{x \mid \mathbf{M}x\}$</td><td>$\lambda x\{x_1\}$</td><td colspan="2" rowspan="2">$\mathbf{V}_1$</td><td></td></tr>
<tr><td>③</td><td colspan="3">$\bigwedge\{x_1 \mid \mathbf{M}x\}$</td><td>②</td></tr>
<tr><td>④</td><td colspan="5">$\bigwedge\{\mathbf{V}x \mid \mathbf{M}x\}$</td><td></td></tr>
<tr><td>⑤</td><td colspan="5">$\forall x\,\{\, \mathbf{M}x \rightarrow \mathbf{V}x \,\}$</td><td></td></tr>
</table>

[4] Alternatively, we informally read

$\bigwedge\{x \mid x \text{ is a man}\}$

as: every x such that x is a man.

Explanation of Computations:

(1) Compositions ① and ② involve function-application, in accordance with *expanded* lambda-conversion. Nothing new here.

(2) Compositions ③ and ④ appeal to the following new composition principle.[5]

λO(∧)	ϕ	$\mathfrak{I} \rightarrow \mathfrak{I}'$
	$\wedge\{\alpha \mid \Phi\}$	$\wedge\mathfrak{I}$
	$\wedge\{\phi(\alpha) \mid \Phi\}$	$\wedge\mathfrak{I}'$

In other words, if $\wedge\{\alpha|\Phi\}$ is a conjunction of items each of which is in the domain of ϕ, then we combine ϕ with $\wedge\{\alpha|\Phi\}$ by applying ϕ to each conjunct and then forming the conjunction of the resulting items.

(3) Item ④ officially reads:

the conjunction of all **V***x* such that **M***x*

This has type ∧S [=S].

(4) The move from ④ to ⑤ is sanctioned by the following new type-logical rule.

$\wedge_\nu\{\Psi \mid \Phi\}$ // $\forall\nu\{\Phi \rightarrow \Psi\}$	[∧-simplification]

Recall that the initial ν is usually omitted when it is obvious which variable is bound, as in the example above, in which case we have the following instance.

$\wedge\{\mathbf{V}x \mid \mathbf{M}x\}$ // $\forall x\{\mathbf{M}x \rightarrow \mathbf{V}x\}$

This rule is derivable from basic principles so long as we also posit one further axiom. First, notice the former asserts

man_1 is **V**, and … , and man_k is **V**

where $\text{man}_1, \ldots, \text{man}_k$ are all the men there are.

The only technical difficulty arises when the latter set is empty, in which case:

$\wedge\{\mathbf{V}x \mid \mathbf{M}x\} = \wedge\varnothing = ?$

So, in order to maintain ∧-simplification, we must also postulate:

$\wedge\varnothing = \text{T}$

which is admittedly strange.

An alternative is to postulate that $\wedge\varnothing = \oslash$, in which case 'every man is virtuous' is not true when there are no men; rather, it is *semantically ill-formed*, being a non-starter due to presupposition-failure. This is not such a preposterous idea. But, for the moment, we employ the usual (modern) logical analysis of 'every' according to which 'every F is G' does not assert, or even presuppose, that there are F's.

[5] This is not the final form of this rule, which is adjusted later.

2. Example 2

every man's mother is kind

every	man	's	mother	[+1]	is	kind
$\lambda P_0 \wedge\{x \mid Px\}$	$\mathbf{M}_0$				$\lambda P_0\{P_1\}$	$\mathbf{K}_0$
$\wedge\{x \mid \mathbf{M}x\}$		$\lambda x\{x_6\}$				
$\wedge\{x_6 \mid \mathbf{M}x\}$			$\lambda x_6\{\mathbf{m}(x)\}$			
$\wedge\{\mathbf{m}(x) \mid \mathbf{M}x\}$				$\lambda x\{x_1\}$		
$\wedge\{\mathbf{m}(x)_1 \mid \mathbf{M}x\}$					$\mathbf{K}_1$	
$\wedge\{\ \mathbf{K}[\mathbf{m}(x)] \mid \mathbf{M}x\ \}$						
$\forall x\ \{\ \mathbf{M}x \rightarrow \mathbf{K}[\mathbf{m}(x)]\ \}$						

3. Example 3

Jay respects every woman

Jay	[+1]	respects	every	woman	[+2]	
J	$\lambda x\{x_1\}$		$\lambda P_0 \wedge\{y \mid Py\}$	$\mathbf{W}_0$		
			$\wedge\{y \mid \mathbf{W}y\}$		$\lambda x\{x_2\}$	
		$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\wedge\{y_2 \mid \mathbf{W}y\}$			
J_1		$\wedge\{\lambda x_1 \mathbf{R}x\mathrm{y} \mid \mathbf{W}y\}$				①
$\wedge\{\ \mathbf{R}\mathrm{J}y \mid \mathbf{W}y\ \}$						②
$\forall y\ \{\ \mathbf{W}y \rightarrow \mathbf{R}\mathrm{J}y\ \}$						

Explanation:

(1) Composition ① appeals to the following composition-rule, which we have already mentioned.

$\lambda O(\wedge)$	ϕ	$\mathfrak{I} \rightarrow \mathfrak{I}'$
	$\wedge\{\alpha \mid \Phi\}$	$\wedge\mathfrak{I}$
	$\wedge\{\phi(\alpha) \mid \Phi\}$	$\wedge\mathfrak{I}'$

(2) Composition ② appeals to the following composition-rule, which is in effect mirror-image.

$\lambda O(\wedge)$	α	$\mathfrak{I}$
	$\wedge\{\phi \mid \Phi\}$	$\wedge(\mathfrak{I} \rightarrow \mathfrak{I}')$
	$\wedge\{\phi(\alpha) \mid \Phi\}$	$\wedge\mathfrak{I}'$

4. **Example 4**

Jay does not respect every woman

	Jay	[+1]	does not	respect	every	woman	[+2]	
	J	$\lambda x\{x_1\}$			$\lambda P_0 \wedge \{y \mid Py\}$	$\mathbf{W}_0$		
					$\wedge\{y \mid \mathbf{W}y\}$		$\lambda x\{x_2\}$	
				$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\wedge\{y_2 \mid \mathbf{W}y\}$			
			$\lambda P_1 \lambda x_1 {\sim} Px$	$\wedge\{\ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\ \}$				
	J_1		$\wedge\{\ \lambda x_1 {\sim}\mathbf{R}xy \mid \mathbf{W}y\ \}$					②
	$\wedge\{\ {\sim}\mathbf{R}Jy \mid \mathbf{W}y\ \}$							
①	※ $\forall y\ \{\ \mathbf{W}y \rightarrow {\sim}\mathbf{R}xy\ \}$ ※							

As indicated by '※' on line ①, this derivation produces an illegitimate reading. The problem is that 'every woman' has narrow-scope in relation to 'not', but the derivation grants 'every woman' wide-scope.[6]

The obvious question then is how do we formally render quantifier-scope restrictions. We propose to do this by placing restraints on junction-composition, which we do *generically* as follows.

$\lambda O(\wedge)$	ϕ $\wedge\{\alpha \mid \Phi\}$	$\mathfrak{I} \rightarrow \mathfrak{I}'$ $\wedge\mathfrak{I}$
	$\wedge\{\phi(\alpha) \mid \Phi\}$	$\wedge\mathfrak{I}'$
	provided $\wedge$ **admits** ϕ	

The next issue is to specify which functions are and are not admitted by $\wedge$. A decent first approximation is obtained as follows.

$\wedge$ admits ϕ if and only if ϕ is **not anti-tonic**

The notion of anti-tonic (a.k.a. monotonic-decreasing) function is described in detail in an appendix. For the moment, suffice it so say that ⟦does not⟧ is a prominent example.

Now, when we go back and examine the computation above, we notice that composition ② involves $\wedge$ *admitting* ⟦does not⟧, which is now officially prohibited. In order to complete the computation, we must instead insert an extra type-logical step, as in the following derivation.

[6] We take 'does not respect' as a logical-compound that is different in meaning from the morphological-compound 'dis-respects'. They are interchangeable in some contexts, but not others.

Jay	[+1]	does not	respect	every	woman	[+2]	
J	$\lambda x\{x_1\}$			$\lambda P_0 \wedge \{y \mid Py\}$	$\mathbf{W}_0$		
				$\wedge\{y \mid \mathbf{W}y\}$		$\lambda x\{x_2\}$	
			$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\wedge\{y_2 \mid \mathbf{W}y\}$			
			$\wedge\{\lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\}$				①
		$\lambda P_1 \lambda x_1 {\sim} Px$	$\lambda x_1 \forall y\{\mathbf{W}y \to \mathbf{R}xy\}$				②
J_1		$\lambda x_1 \ {\sim}\forall y\{\mathbf{W}y \to \mathbf{R}xy\}$					
${\sim}\forall y \{ \mathbf{W}y \to \mathbf{R}Jy \}$							

Note, in particular, that ② is obtained from ① by the following type-logical derivation.

(1)	$\wedge\{\lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\}$	$\wedge(D_1 \to S)$	1	Pr
(2)	x_1	D_1	2	As
(3)	$\wedge\{\mathbf{R}xy \mid \mathbf{W}y\}$	$\wedge S$ [=S]	12	1,2,λO(∧)
(4)	$\forall y\{\mathbf{W}y \to \mathbf{R}xy\}$	S	12	∧-Simp
(5)	$\lambda x_1 \forall y\{\mathbf{W}y \to \mathbf{R}xy\}$	$D_1 \to S$	1	2,4,λI

Moreover, item ② readily combines with ⟦does not⟧ using expanded lambda-conversion.

5. Example 5

every man respects every woman

	every	man	[+1]	respects	every	woman	[+2]	
				$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\wedge\{y_2 \mid \mathbf{W}y\}$			
	$\wedge\{x_1 \mid \mathbf{M}x\}$			$\wedge\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$				
①	$\wedge\{ \wedge\{ \mathbf{R}xy \mid \mathbf{W}y \} \mid \mathbf{M}x \}$							
	$\forall x \{ \mathbf{M}x \to \forall y \{\mathbf{W}y \to \mathbf{R}xy\} \}$							

	every	man	[+1]	respects	every	woman	[+2]	
				$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\wedge\{y_2 \mid \mathbf{W}y\}$			
	$\wedge\{x_1 \mid \mathbf{M}x\}$			$\wedge\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$				
②	$\wedge\{ \wedge\{ \mathbf{R}xy \mid \mathbf{M}x \} \mid \mathbf{W}y \}$							
	$\forall y \{ \mathbf{W}y \to \forall x \{\mathbf{M}x \to \mathbf{R}xy\} \}$							

Compositions ① and ② appeal, respectively, to the following composition-rules.

$\wedge\{\alpha \mid \Phi\}$	$\wedge(\mathfrak{I})$
$\wedge\{\phi \mid \Psi\}$	$\wedge(\mathfrak{I} \to \mathfrak{I}')$
$\wedge\{\wedge\{\phi(\alpha) \mid \Psi\} \mid \Phi\}$	$\wedge(\mathfrak{I}')$

$\bigwedge\{\alpha \mid \Phi\}$ $\bigwedge\{\phi \mid \Psi\}$	$\bigwedge(\Im)$ $\bigwedge(\Im \rightarrow \Im')$
$\bigwedge\{\bigwedge\{\phi(\alpha) \mid \Phi\} \mid \Psi\}$	$\bigwedge(\Im')$

6. Example 5

every man who respects every man's father respects every man's mother

every	man who respects every man's father	[+1]	respects	every	man	's	mother	[+2]
				$\lambda P_0 \bigwedge\{x \mid Px\}$	$\mathbf{M}_0$			
				$\bigwedge\{x \mid \mathbf{M}x\}$		$\lambda x\{x_6\}$		
				$\bigwedge\{x_6 \mid \mathbf{M}x\}$			$\lambda x_6\{\mathbf{m}(x)\}$	
				$\bigwedge\{\mathbf{m}(x) \mid \mathbf{M}x\}$				$\lambda x\{x_2\}$
			$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\bigwedge\{\mathbf{m}(x)_2 \mid \mathbf{M}x\}$				
$\lambda P_0 \bigwedge\{x \mid Px\}$	(insert output of tablebelow)							
$\bigwedge\{ x \mid \mathbf{M}x \ \& \ \forall y \{ \mathbf{M}y \rightarrow \mathbf{R}[x,\mathbf{f}(y)] \} \}$		$\lambda x\{x_1\}$						
$\bigwedge\{ x_1 \mid \mathbf{M}x \ \& \ \forall y \{ \mathbf{M}y \rightarrow \mathbf{R}[x,\mathbf{f}(y)] \} \}$			$\bigwedge\{\lambda x_1 \mathbf{R}[x,\mathbf{m}(y)] \mid \mathbf{M}y\}$					
$\bigwedge\{ \bigwedge\{\mathbf{R}[x,\mathbf{m}(y)] \mid \mathbf{M}y\} \mid \mathbf{M}x \ \& \ \forall y \{ \mathbf{M}y \rightarrow \mathbf{R}[x,\mathbf{f}(y)] \} \}$								
$\forall x \{ \{\mathbf{M}x \ \& \ \forall y \{ \mathbf{M}y \rightarrow \mathbf{R}[x,\mathbf{f}(y)] \}\} \rightarrow \forall y \{ \mathbf{M}y \rightarrow \mathbf{R}[x,\mathbf{m}(y)] \} \}$								

man	who$_1$	respects	every	man	's	father	[+2]
			$\lambda P_0 \bigwedge\{x \mid Px\}$	$\mathbf{M}_0$			
			$\bigwedge\{x \mid \mathbf{M}x\}$		$\lambda x\{x_6\}$		
			$\bigwedge\{x_6 \mid \mathbf{M}x\}$			$\lambda x_6\{\mathbf{f}(x)\}$	
			$\bigwedge\{\mathbf{f}(x) \mid \mathbf{M}x\}$				$\lambda x\{x_2\}$
		$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\bigwedge\{\mathbf{f}(x)_2 \mid \mathbf{M}x\}$				
		$\bigwedge\{ \lambda x_1 \mathbf{R}[x,\mathbf{f}(y)] \mid \mathbf{M}y \}$					
	$\lambda Q_1 \lambda P_0 \lambda x_0\{Px \& Qx\}$	$\lambda x_1 \ \forall y \{ \mathbf{M}y \rightarrow \mathbf{R}[x,\mathbf{f}(y)] \}$					
$\mathbf{M}_0$	$\lambda P_0 \ \lambda x_0 \{ Px \ \& \ \forall y \{ \mathbf{M}y \rightarrow \mathbf{R}[x,\mathbf{f}(y)] \}\}$						
$\lambda x_0 \{ \mathbf{M}x \ \& \ \forall y \{ \mathbf{M}y \rightarrow \mathbf{R}[x,\mathbf{f}(y)] \}\}$							