1. A New Theory of Quantifiers

In what follows, we offer an account of quantifiers in terms of a new class of type-theoretic objects, which we call ***junctions***, which are infinitary-operators.[1] Before we are finished, we will have proposed six different junctions.

$\bigwedge$	conjunction	every
$\bigvee$	disjunction	some
Л	subjunction	any
⊠	exclusive-disjunction	exactly-one
Σ	sum	"a" (base)[2]
Π	product	"a" (promoted)

2. Infinitary-Conjunction

First, we posit a new type-forming operator $\bigwedge$ – called 'conjunction', defined so that

if $\mathfrak{I}$ is a type, then so is $\bigwedge\mathfrak{I}$.

We further propose the following type-identity,

$\bigwedge \mathrm{S} = \mathrm{S}$

which reflects the intuition that the conjunction of any collection of sentences is itself a sentence. We further propose the following formation rule.

If ν is a variable, ε is an expression of type $\mathfrak{I}$, and Φ is a formula, then

$\bigwedge_{\nu}\{\varepsilon \mid \Phi\}$

is an expression of type $\bigwedge\mathfrak{I}$, which is read:

the conjunction (over ν) of all ε such that Φ.

If ε and Φ share exactly one free variable, then this is understood to be the variable of abstraction, so we omit it, in which case we have (informal) expressions of the following form,

$\bigwedge\{\varepsilon \mid \Phi\}$

which we read:

the conjunction of all ε such that Φ.

With this in hand, we interpret 'every' as follows.

$$[\![\text{every}]\!] = \lambda P_0 \bigwedge\{x \mid Px\}$$

For example:

$[\![\text{every man}]\!] = \bigwedge\{x \mid x \text{ is a man}\}$
$=$ the conjunction of all men
$\approx$ man_1 and man_2 and … and man_k

where $\{\text{man}_1, \ldots, \text{man}_k\}$ constitute all the men.

[1] By an infinitary-operator, we mean an operator that acts on *a collection of arguments* of arbitrary size, even infinite, and even empty.

[2] The indefinite article 'a' is scare-quoted because the indefinite article is often not pronounced; in English, it is required before a singular-noun, but prohibited before a count-noun or mass-noun. Compare this to Spanish which has plural indefinite articles ['unas' and 'unos'].

3. Examples

1. every man is virtuous

	every	man	[+1]	is	virtuous	
	$\lambda P_0 \bigwedge\{x \mid Px\}$	$\mathbf{M}_0$		$\lambda P_0\{P_1\}$	$\mathbf{V}_0$	
①	$\bigwedge\{x \mid \mathbf{M}x\}$		$\lambda x\{x_1\}$			
③	$\bigwedge\{x_1 \mid \mathbf{M}x\}$			$\mathbf{V}_1$		②
④	$\bigwedge\{\mathbf{V}x \mid \mathbf{M}x\}$					
⑤	$\forall x\ \{\ \mathbf{M}x \rightarrow \mathbf{V}x\ \}$					

(1) Compositions ① and ② involve function-application, in accordance with *expanded* lambda-conversion. Nothing new here.

(2) Compositions ③ and ④ appeal to the following new composition principle.[3]

$\bigwedge$-composition	
α	α is any expression, or null
$\bigwedge_v\{\beta \mid \Phi\}$	
$\{\alpha,\beta\} \vdash \gamma$	a sub-derivation of γ from $\{\alpha,\beta\}$
$\bigwedge_v\{\gamma \mid \Phi\}$	

(4) The move from ④ to ⑤ is sanctioned by the following new type-logical rule.

$$\bigwedge_v\{\Psi \mid \Phi\} \ / \ \forall v\{\Phi \rightarrow \Psi\} \qquad [\bigwedge\text{-simplification}]$$

2. every man's mother is kind

every	man	's	mother	[+1]	is	kind
$\lambda P_0 \bigwedge\{x \mid Px\}$	$\mathbf{M}_0$				$\lambda P_0\{P_1\}$	$\mathbf{K}_0$
$\bigwedge\{x \mid \mathbf{M}x\}$		$\lambda x\{x_6\}$				
$\bigwedge\{x_6 \mid \mathbf{M}x\}$			$\lambda x_6\{\mathbf{m}(x)\}$			
$\bigwedge\{\mathbf{m}(x) \mid \mathbf{M}x\}$				$\lambda x\{x_1\}$		
$\bigwedge\{\mathbf{m}(x)_1 \mid \mathbf{M}x\}$					$\mathbf{K}_1$	
$\bigwedge\{\ \mathbf{K}[\mathbf{m}(x)] \mid \mathbf{M}x\ \}$						
$\forall x\ \{\ \mathbf{M}x \rightarrow \mathbf{K}[\mathbf{m}(x)]\ \}$						

[3] This is not the final form of this rule, which is adjusted later.

3. Jay respects every woman

<table>
<tr><td>Jay</td><td>[+1]</td><td>respects</td><td>every</td><td>woman</td><td>[+2]</td></tr>
<tr><td>J</td><td>$\lambda x\{x_1\}$</td><td></td><td>$\lambda P_0\wedge\{y \mid Py\}$</td><td>$\mathbf{W}_0$</td><td></td></tr>
<tr><td></td><td></td><td></td><td colspan="2">$\wedge\{y \mid \mathbf{W}y\}$</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td></td><td></td><td>$\lambda y_2\lambda x_1\mathbf{R}xy$</td><td colspan="3">$\wedge\{y_2 \mid \mathbf{W}y\}$</td></tr>
<tr><td colspan="2">J_1</td><td colspan="4">$\wedge\{\lambda x_1\mathbf{R}xy \mid \mathbf{W}y\}$</td></tr>
<tr><td colspan="6">$\wedge\{\ \mathbf{R}Jy \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="6">$\forall y\ \{\ \mathbf{W}y \rightarrow \mathbf{R}Jy\ \}$</td></tr>
</table>

4. Jay does not respect every woman

<table>
<tr><td></td><td>Jay</td><td>[+1]</td><td>does-not</td><td>respect</td><td>every</td><td>woman</td><td>[+2]</td><td></td></tr>
<tr><td></td><td>J</td><td>$\lambda x\{x_1\}$</td><td></td><td></td><td>$\lambda P_0\wedge\{y \mid Py\}$</td><td>$\mathbf{W}_0$</td><td></td><td></td></tr>
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<tr><td></td><td></td><td></td><td></td><td>$\lambda y_2\lambda x_1\mathbf{R}xy$</td><td colspan="3">$\wedge\{y_2 \mid \mathbf{W}y\}$</td><td></td></tr>
<tr><td></td><td></td><td></td><td>$\lambda P_1\lambda x_1{\sim}Px$</td><td colspan="4">$\wedge\{\ \lambda x_1\mathbf{R}xy \mid \mathbf{W}y\ \}$</td><td></td></tr>
<tr><td></td><td colspan="2">J_1</td><td colspan="5">$\wedge\{\ \lambda x_1{\sim}\mathbf{R}xy \mid \mathbf{W}y\ \}$</td><td>②</td></tr>
<tr><td></td><td colspan="7">$\wedge\{\ {\sim}\mathbf{R}Jy \mid \mathbf{W}y\ \}$</td><td></td></tr>
<tr><td>①</td><td colspan="7">※ $\forall y\ \{\ \mathbf{W}y \rightarrow {\sim}\mathbf{R}xy\ \}$ ※</td><td></td></tr>
</table>

As indicated by '※' on line ①, this derivation produces an illegitimate reading. The problem is that 'every woman' has *narrow-scope* in relation to 'not', but the derivation grants 'every woman' *wide-scope*.[4]

The obvious question then is how do we formally render quantifier-scope restrictions. We propose to do this by placing restraints on junction-composition, which we do as follows.

α	α is any expression, or null
$\wedge_v\{\beta \mid \Phi\}$	$\wedge$ ***admits*** α
$\alpha,\beta \vdash \gamma$	a sub-derivation of γ from $\{\alpha,\beta\}$
$\wedge_v\{\gamma \mid \Phi\}$	

This leaves the following outstanding theoretical question: how do we specify which items are and are not admitted by $\wedge$? We do so by constructing a list of dis-admitted items.[5] We start the list as follows.[6]

---

[4] We take 'does not respect' as a logical-compound that is different in meaning from the morphological-compound 'dis-respects'. They are interchangeable in some contexts, but not others.

[5] We presume that if an item is not explicitly dis-admitted, then it is automatically admitted.

[6] *Ideally*, we will be able to specify what underlying semantic feature, or features, the dis-admitted items share, in virtue of which they are dis-admitted. We might fall short of this ideal, however, in which case we will have to accept a certain amount of theoretical inelegance in our system.

(∧1)	∧ does not admit $\lambda P_1\lambda x_1{\sim}Px$ (‘does-not’).

Now, when we go back and examine the computation above, we notice that composition ② involves ∧ *admitting* ⟦does not⟧, which is now officially prohibited. So, in order to complete the computation, we must instead insert an extra type-logical step, as in the following derivation.

<table>
<tr><td>Jay</td><td>[+1]</td><td>does-not</td><td>respect</td><td>every</td><td>woman</td><td>[+2]</td><td></td></tr>
<tr><td>J</td><td>$\lambda x\{x_1\}$</td><td></td><td></td><td>$\lambda P_0\wedge\{y \mid Py\}$</td><td>$\mathbf{W}_0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td colspan="2">$\wedge\{y \mid \mathbf{W}y\}$</td><td>$\lambda x\{x_2\}$</td><td></td></tr>
<tr><td></td><td></td><td></td><td>$\lambda y_2\lambda x_1\mathbf{R}xy$</td><td colspan="3">$\wedge\{y_2 \mid \mathbf{W}y\}$</td><td></td></tr>
<tr><td></td><td></td><td></td><td colspan="4">$\wedge\{\lambda x_1\mathbf{R}xy \mid \mathbf{W}y\}$</td><td>①</td></tr>
<tr><td></td><td></td><td>$\lambda P_1\lambda x_1{\sim}Px$</td><td colspan="4">$\lambda x_1\forall y\{\mathbf{W}y \rightarrow \mathbf{R}xy\}$</td><td>②</td></tr>
<tr><td colspan="2">J_1</td><td colspan="5">$\lambda x_1 \,{\sim}\forall y\{\mathbf{W}y \rightarrow \mathbf{R}xy\}$</td><td></td></tr>
<tr><td colspan="7">${\sim}\forall y \{ \mathbf{W}y \rightarrow \mathbf{R}Jy \}$</td><td></td></tr>
</table>

In particular, ② is obtained from ① by the following type-logical derivation.

(1)	$\wedge\{\lambda x_1\mathbf{R}xy \mid \mathbf{W}y\}$	$\wedge(D_1\rightarrow S)$	1	Pr
(2)	x_1	D_1	2	As
(3)	$\wedge\{\mathbf{R}xy \mid \mathbf{W}y\}$	$\wedge S$ [=S]	12	1,2,∧-com[7]
(4)	$\forall y\{\mathbf{W}y \rightarrow \mathbf{R}xy\}$	S	12	∧-Simp
(5)	$\lambda x_1\forall y\{\mathbf{W}y \rightarrow \mathbf{R}xy\}$	$D_1\rightarrow S$	1	2,4,λI

Moreover, item ② readily combines with ⟦does not⟧ using expanded lambda-conversion.

5. every man respects every woman

<table>
<tr><td></td><td>every</td><td>man</td><td>[+1]</td><td>respects</td><td>every</td><td>woman</td><td>[+2]</td></tr>
<tr><td></td><td colspan="3" rowspan="2">$\wedge\{x_1 \mid \mathbf{M}x\}$</td><td>$\lambda y_2\lambda x_1\mathbf{R}xy$</td><td colspan="3">$\wedge\{y_2 \mid \mathbf{W}y\}$</td></tr>
<tr><td></td><td colspan="4">$\wedge\{ \lambda x_1\mathbf{R}xy \mid \mathbf{W}y \}$</td></tr>
<tr><td>①</td><td colspan="7">$\wedge\{ \wedge\{ \mathbf{R}xy \mid \mathbf{W}y \} \mid \mathbf{M}x \}$</td></tr>
<tr><td></td><td colspan="7">$\forall x \{ \mathbf{M}x \rightarrow \forall y \{\mathbf{W}y \rightarrow \mathbf{R}xy\} \}$</td></tr>
</table>

<table>
<tr><td></td><td>every</td><td>man</td><td>[+1]</td><td>respects</td><td>every</td><td>woman</td><td>[+2]</td></tr>
<tr><td></td><td colspan="3" rowspan="2">$\wedge\{x_1 \mid \mathbf{M}x\}$</td><td>$\lambda y_2\lambda x_1\mathbf{R}xy$</td><td colspan="3">$\wedge\{y_2 \mid \mathbf{W}y\}$</td></tr>
<tr><td></td><td colspan="4">$\wedge\{ \lambda x_1\mathbf{R}xy \mid \mathbf{W}y \}$</td></tr>
<tr><td>②</td><td colspan="7">$\wedge\{ \wedge\{ \mathbf{R}xy \mid \mathbf{M}x \} \mid \mathbf{W}y \}$</td></tr>
<tr><td></td><td colspan="7">$\forall y \{ \mathbf{W}y \rightarrow \forall x \{\mathbf{M}x \rightarrow \mathbf{R}xy\} \}$</td></tr>
</table>

[7] Officially, ∧-com requires a sub-derivation; in most applications, this sub-derivation is an earlier theorem. In this particular case, the earlier theorem is completely trivial – $\{\lambda x_1 \mathrm{R}xy , x_1\} \vdash \mathrm{R}xy$ – and is accordingly not even mentioned. Other times, it is not completely trivial, and is accordingly referred to as an earlier theorem.

Compositions ① and ② involve an iterated application of ∧-composition, as detailed in the following derivations.

(1)	$\wedge\{ x_1 \mid \mathbf{M}x \}$	∧S	1	P
(2)	$\wedge\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$	∧S	2	P
(3)	$\wedge\{ x_1 \times \wedge\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \} \mid \mathbf{M}x \}$	∧∧S	12	1,2,∧C
(4)	$x_1 \times \wedge\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \} \vdash \wedge\{ \mathbf{R}xy \mid \mathbf{W}y \}$		∅	ET
(5)	$\wedge\{ \wedge\{ \mathbf{R}xy \mid \mathbf{W}y \} \mid \mathbf{W}y \} \mid \mathbf{M}x \}$	∧∧S	12	3,4,∧C

(1)	$\wedge\{ x_1 \mid \mathbf{M}x \}$	∧S	1	P
(2)	$\wedge\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$	∧S	2	P
(3)	$\wedge\{ x_1 \times \wedge\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{M}x \} \mid \mathbf{W}y \}$	∧∧S	12	1,2,∧C
(4)	$x_1 \times \wedge\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{M}x \} \vdash \wedge\{ \mathbf{R}xy \mid \mathbf{M}x \}$		∅	ET
(5)	$\wedge\{ \wedge\{ \mathbf{R}xy \mid \mathbf{W}y \} \mid \mathbf{M}x \} \mid \mathbf{W}y \}$	∧∧S	12	3,4,∧C

6. every man who respects every man's father respects every man's mother

every	man who respects every man's father	[+1]	respects	every	man	's	mother	[+2]
				$\lambda P_0 \wedge\{x \mid Px\}$	$\mathbf{M}_0$			
				$\wedge\{x \mid \mathbf{M}x\}$		$\lambda x\{x_6\}$		
				$\wedge\{x_6 \mid \mathbf{M}x\}$			$\lambda x_6\{\mathbf{m}(x)\}$	
				$\wedge\{\mathbf{m}(x) \mid \mathbf{M}x\}$				$\lambda x\{x_2\}$
			$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\wedge\{\mathbf{m}(x)_2 \mid \mathbf{M}x\}$				
$\lambda P_0 \wedge\{x \mid Px\}$	(insert output of tablebelow)							
$\wedge\{ x \mid \mathbf{M}x \,\&\, \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{f}(y)] \} \}$		$\lambda x\{x_1\}$						
$\wedge\{ x_1 \mid \mathbf{M}x \,\&\, \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{f}(y)] \} \}$			$\wedge\{\lambda x_1 \mathbf{R}[x,\mathbf{m}(y)] \mid \mathbf{M}y\}$					
$\wedge\{ \wedge\{\mathbf{R}[x,\mathbf{m}(y)] \mid \mathbf{M}y\} \mid \mathbf{M}x \,\&\, \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{f}(y)] \} \}$								
$\forall x \{ \{\mathbf{M}x \,\&\, \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{f}(y)] \}\} \to \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{m}(y)] \} \}$								

man	who	[+1]	respects	every	man	's	father	[+2]	
	$\lambda Q_1 \lambda P_0 \lambda x_0 \{Px \& Qx\}$	$\lambda x\{x_1\}$		$\lambda P_0 \wedge\{x \mid Px\}$	$\mathbf{M}_0$				
				$\wedge\{x \mid \mathbf{M}x\}$		$\lambda x\{x_6\}$			
				$\wedge\{x_6 \mid \mathbf{M}x\}$			$\lambda x_6\{\mathbf{f}(x)\}$		
				$\wedge\{ \mathbf{f}(x) \mid \mathbf{M}x \}$				$\lambda x\{x_2\}$	
			$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\wedge\{ \mathbf{f}(x)_2 \mid \mathbf{M}x \}$					
			$\wedge\{ \lambda x_1 \mathbf{R}[x,\mathbf{f}(x)] \mid \mathbf{M}x \}$						①
	$\lambda Q_1\, \lambda P_0\, \lambda x_0 \{ Px \,\&\, Qx \}$		$\lambda x_1\, \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{f}(y)] \}$						②
$\mathbf{M}_0$	$\lambda P_0\, \lambda x_0 \{ Px \,\&\, \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{f}(y)] \} \}$								
$\lambda x_0 \{ \mathbf{M}x \,\&\, \forall y \{ \mathbf{M}y \to \mathbf{R}[x,\mathbf{f}(y)] \} \}$									

Note that ① could be combined with ⟦who$_1$⟧, using ⋀-composition, *except* that we now add the following further restriction on admissibility.

(⋀2)	⋀ does not admit any relative-pronoun-phrase.[8]

This corresponds to a syntactic-restriction on quantifier-movement, according to which 'every' cannot be moved outside any relative clause in which it appears.[9]

Given that ⋀ does not admit relative-pronoun-phrases, we must instead convert ① into ②, using a type-logical transformation, the details of which are left as an exercise.

4. Existential Quantifiers – Infinitary-Disjunction

In the earlier example,

every man respects every woman

the respective scopes of the two quantifier-phrases is irrelevant to the final computation, since the two resulting formulas are logically equivalent. By contrast, in the following example,

every man respects **some** woman

the scope-ambiguity produces two logically-different readings.

In order to see this, we now offer our new account of 'some', which involves positing a second junction – ⋁, (infinitary) **disjunction** – which is parallel to (infinitary) conjunction. First, we propose a new type-forming operator, defined so that

if $\mathfrak{I}$ is a type, then so is $\bigvee\mathfrak{I}$

As with ⋀, we also add the following expressions to type-theory

$\bigvee_{\nu}\{\varepsilon \mid \Phi\}$

which we read as:

the disjunction [over ν] of all ε such that Φ

As with ⋀, if ε and Φ share exactly one free variable, then ν is omitted, and we have (informal) expressions of the form.

$\bigvee\{\varepsilon \mid \Phi\}$

With this in hand, we interpret 'every' as follows.

⟦some⟧	=	$\lambda P_0 \bigvee\{x \mid Px\}$

For example:

⟦some man⟧	=	$\bigvee\{x \mid x$ is a man$\}$
	=	the disjunction of all men
	≈	man_1 or man_2 or … or man_k

where $\{\text{man}_1, \ldots, \text{man}_k\}$ constitute all the men.

---

[8] In other words, an NP headed by a relative pronoun, such as 'who' and 'whose mother'.

[9] We now have two items not admitted by ⋀; unfortunately, there is no obvious systematic semantic connection between ⟦does not⟧ and ⟦who⟧. We need to work on this.

1. Examples

1. Jay does not respect some woman

Jay	[+1]	does-not	respect	some	woman	[+2]
J	$\lambda x\{x_1\}$			$\lambda P_0 \vee \{y \mid Py\}$	$\mathbf{W}_0$	
				$\vee\{y \mid \mathbf{W}y\}$		$\lambda x\{x_2\}$
			$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\vee\{y_2 \mid \mathbf{W}y\}$		
		$\lambda P_1 \lambda x_1\{\sim Px\}$	$\vee\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$			
J_1		$\vee\{ \lambda x_1 {\sim} \mathbf{R}xy \mid \mathbf{W}y \}$				
$\vee\{ \sim \mathbf{R}\mathrm{J}y \mid \mathbf{W}y \}$						
$\exists y \{ \mathbf{W}y \ \& \sim \mathbf{R}xy \}$						

The computations parallel those for $\wedge$. Rather than repeat these rules, we instead offer the following more general version[10]

Junction-Composition (Ж-Com)	
α	α is any expression (**or null**)
Ж$_\nu\{\beta \mid \Phi\}$	Ж admits α
$\{\alpha,\beta\} \vdash \gamma$	a sub-derivation of γ from $\{\alpha,\beta\}$
Ж$_\nu\{\gamma \mid \Phi\}$	

Here, α, β, γ are expressions (even junctions[11]), and Ж[12] is any junction.

Also, note that the last step appeals to the following simplification principle.

$$\vee_\nu\{\Psi \mid \Phi\} \ / \ \exists\nu\{\Phi \& \Psi\} \qquad [\vee\text{-simplification}]$$

2. every man respects some woman

'every man' has wide scope

every man $_1$	respects	some woman $_2$
	$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\vee\{ y_2 \mid \mathbf{W}y \}$
$\wedge\{ x_1 \mid \mathbf{M}x \}$	$\vee\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$	
$\wedge\{ \vee\{ \mathbf{R}xy \mid \mathbf{W}y \} \mid \mathbf{M}x \}$		
$\forall x \{ \mathbf{M}x \rightarrow \exists y\{\mathbf{W}y \ \& \ \mathbf{R}xy\} \}$		

'some woman' has wide scope

every man $_1$	respects	some woman $_2$
	$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\vee\{ y_2 \mid \mathbf{W}y \}$
$\wedge\{ x_1 \mid \mathbf{M}x \}$	$\vee\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$	
$\vee\{ \wedge\{ \mathbf{R}xy \mid \mathbf{M}x \} \mid \mathbf{W}y \}$		
$\exists y\{ \mathbf{W}y \ \& \ \forall x \{\mathbf{M}x \rightarrow \mathbf{R}xy\} \}$		

Note, in particular, that the scope-ambiguity is inherent in the rules of composition,[13] so long as both $\wedge$ and $\vee$ admit each other, which we hereby postulate.[14]

[10] This is still not quite the final word on junction-composition.
[11] We accept the ambiguity between the use of 'junction' to refer to the operator, and to refer to the result of applying the operator; this is consonant with the same ambiguity in the word 'conjunction'.
[12] 'Ж' is Cyrillic letter "zhe", which is a soft 'J' (IPA: ?); there is no hard 'J' (IPA: ?) sound in Russian.

5. ‘No’ [Universal-Negative]

We have redesigned ⟦every⟧ and ⟦some⟧. Next on the list is ⟦no⟧, which we formalize as follows, using the Medieval idea that ‘no’ is a combination of *universal* and *negative*.

$$⟦\text{no}⟧ \quad = \quad \lambda P_0 \wedge\{\lambda Q{\sim}Qx \mid Px\}$$

For example,

$$⟦\text{no man}⟧ \quad = \quad \wedge\{\lambda Q{\sim}Qx \mid \mathbf{M}x\}$$

Notice that $\lambda Q{\sim}Qx$ corresponds to "not x", so intuitively, ⟦no man⟧ corresponds to the following conjunction

not-man$_1$ and not-man$_2$ and … and not-man$_k$

where $\{\text{man}_1, \ldots, \text{man}_k\}$ constitute all the men.

1. no man is virtuous

<table>
<tr><td>no</td><td>man</td><td>[+1]</td><td>is</td><td>virtuous</td></tr>
<tr><td>$\lambda P_0\wedge\{\lambda Q{\sim}Qx \mid Px\}$</td><td>$\mathbf{M}_0$</td><td></td><td>$\lambda P_0\{P_1\}$</td><td>$\mathbf{V}_0$</td></tr>
<tr><td colspan="2">$\wedge\{ \lambda Q{\sim}Qx \mid \mathbf{M}x \}$</td><td>$\lambda x\{x_1\}$</td><td colspan="2"></td></tr>
<tr><td colspan="3">$\wedge\{ \lambda Q_1{\sim}Qx \mid \mathbf{M}x \}$</td><td colspan="2">$\mathbf{V}_1$</td></tr>
<tr><td colspan="5">$\wedge\{ {\sim}\mathbf{V}x \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="5">$\forall x \{ \mathbf{M}x \to {\sim}\mathbf{V}x \}$
${\sim}\exists x \{ \mathbf{M}x \;\&\; \mathbf{V}x \}$</td></tr>
</table>

2. no man respects every woman

<table>
<tr><td>no</td><td>man</td><td>[+1]</td><td>respects</td><td>every</td><td>woman</td><td>[+2]</td><td></td></tr>
<tr><td>$\lambda P_0\wedge\{\lambda Q{\sim}Qx \mid Px\}$</td><td>$\mathbf{M}_0$</td><td></td><td rowspan="2"></td><td>$\lambda P_0\wedge\{y \mid Py\}$</td><td>$\mathbf{W}_0$</td><td></td><td></td></tr>
<tr><td colspan="2">$\wedge\{ \lambda Q{\sim}Qx \mid \mathbf{M}x \}$</td><td>$\lambda x\{x_1\}$</td><td colspan="2">$\wedge\{ y \mid \mathbf{W}y \}$</td><td>$\lambda x\{x_2\}$</td><td></td></tr>
<tr><td colspan="3" rowspan="3">$\wedge\{ \lambda Q_1{\sim}Qx \mid \mathbf{M}x \}$</td><td>$\lambda y_2\lambda x_1\mathbf{R}xy$</td><td colspan="3">$\wedge\{ y_2 \mid \mathbf{W}y \}$</td><td></td></tr>
<tr><td colspan="4">$\wedge\{ \lambda x_1\mathbf{R}xy \mid \mathbf{W}y \}$</td><td>①</td></tr>
<tr><td colspan="4">$\lambda x_1\ \forall y\{\mathbf{W}y \to \mathbf{R}xy\}$</td><td>②</td></tr>
<tr><td colspan="7">$\wedge\{ {\sim}\forall y\{\mathbf{W}y \to \mathbf{R}xy\} \mid \mathbf{M}x \}$</td><td></td></tr>
<tr><td colspan="7">$\forall x \{ \mathbf{M}x \to {\sim}\forall y\{\mathbf{W}y \to \mathbf{R}xy\} \}$</td><td></td></tr>
</table>

In the above derivation, the derivation of ② from ① side-steps the following further restriction on admissibility.

(∧3) ∧ does not admit $\wedge\{ \lambda Q{\sim}Qx \mid Px \}$ (‘no P’).

This restriction theoretically underwrites the intuition that there is no reading of this sentence that accords wide-scope to ‘every woman’.

[13] So, in particular, this ambiguity is neither lexical nor structural, but rather compositional.
[14] Officially, we only have to postulate cases of dis-admission, so this postulate is technically superfluous.