

## 1. New Theory of Quantifiers

We propose an account of quantifiers in terms of *junctions*, which are infinitary-operators.<sup>1</sup> All told (or perhaps, all *tolled*), we will propose six different junctions, as follows.

$\wedge$	conjunction	every
$\vee$	disjunction	some
$\prod$	subjunction	any
$\boxtimes$	exclusive-disjunction	exactly-one
$\Sigma$	sum	"a" (base) <sup>2</sup>
$\Pi$	product	"a" (promoted)

## 2. General Formation Rules for Junctions

First, for each junction  $\mathcal{K}$ , we posit a new type-forming operator  $\mathcal{K}$ , defined so that if  $\mathfrak{S}$  is a type, then so is  $\mathcal{K}\mathfrak{S}$ .

We further propose the following formation rule.

If  $v$  is a variable,  $\varepsilon$  is an expression of type  $\mathfrak{S}$ , and  $\Phi$  is a formula, then

$$\mathcal{K}_v\{\varepsilon \mid \Phi\}$$

is an expression of type  $\mathcal{K}\mathfrak{S}$ , which we read as:

the  $\mathcal{K}$  (over  $v$ ) of all  $\varepsilon$  such that  $\Phi$

where ‘ $\mathcal{K}$ ’ is replaced by the appropriate term (e.g., ‘conjunction’ or ‘product’).

If  $\varepsilon$  and  $\Phi$  share exactly-one free variable, then this is understood to be the variable of abstraction, so we omit it, in which case we have (informal) expressions of the following form,

$$\mathcal{K}\{\varepsilon \mid \Phi\}$$

which we read as:

the  $\mathcal{K}$  of all  $\varepsilon$  such that  $\Phi$ .

## 3. Interpretation of Quantifiers (so far)

$\llbracket \text{every} \rrbracket$	=	$\lambda P_0 \wedge \{x \mid Px\}$
--------------------------------------	---	------------------------------------

$\llbracket \text{some} \rrbracket$	=	$\lambda P_0 \vee \{x \mid Px\}$
-------------------------------------	---	----------------------------------

$\llbracket \text{no} \rrbracket$	=	$\lambda P_0 \wedge \{\lambda Q \sim Qx \mid Px\}$
-----------------------------------	---	--

<sup>1</sup> By an infinitary-operator, we mean an operator that acts on a *collection of arguments* of arbitrary size, even infinite, and even empty.

<sup>2</sup> The word ‘a’ is scare-quoted because it is often not pronounced. In English, singular common-noun-phrases *require* ‘a’, but plural-phrases and mass-phrases *prohibit* ‘a’. By comparison, Spanish has plural forms of ‘a’ [‘unas’ and ‘unos’].

## 4. Composition (so far)

Junction-Composition ( $\mathcal{K}$ -Com)	
$\alpha$	$\alpha$ is any expression ( <b>or null</b> )
$\mathcal{K}_v\{\beta \mid \Phi\}$	$\mathcal{K}$ admits $\alpha$
$\{\alpha, \beta\} \vdash \gamma$	a sub-derivation of $\gamma$ from $\{\alpha, \beta\}$
$\mathcal{K}_v\{\gamma \mid \Phi\}$	

Here,  $\alpha, \beta, \gamma$  are expressions (even junctions<sup>3</sup>), and  $\mathcal{K}$  is any junction.

## 5. Admissibility-Restrictions (so far)

( $\wedge_1$ )  $\wedge$  does not admit  $\lambda P_1 \lambda x_1 \sim P x$  ('does-not').

( $\wedge_2$ )  $\wedge$  does not admit any relative-pronoun-phrase.<sup>4</sup>

( $\wedge_3$ )  $\wedge$  does not admit  $\wedge\{\lambda Q \sim Q x \mid P x\}$  ('no P').

## 6. Simplification Principles (so far)

$\wedge_v\{\Psi \mid \Phi\} / \forall v\{\Phi \rightarrow \Psi\}$  [ $\wedge$ -simplification]

$\vee_v\{\Psi \mid \Phi\} / \exists v\{\Phi \& \Psi\}$  [ $\vee$ -simplification]

## 7. Examples

- every man's mother is kind

every	man	's	mother	[+1]	is	kind
$\lambda P_0 \wedge\{x \mid P x\}$	$\mathbf{M}_0$				$\lambda P_0\{P_1\}$	$\mathbf{K}_0$
$\wedge\{x \mid \mathbf{M}x\}$		$\lambda x\{x_6\}$				
	$\wedge\{x_6 \mid \mathbf{M}x\}$		$\lambda x_6\{\mathbf{m}(x)\}$			
		$\wedge\{\mathbf{m}(x) \mid \mathbf{M}x\}$		$\lambda x\{x_1\}$		
			$\wedge\{\mathbf{m}(x)_1 \mid \mathbf{M}x\}$			$\mathbf{K}_1$
					$\wedge\{\mathbf{K}[\mathbf{m}(x)] \mid \mathbf{M}x\}$	
					$\forall x\{\mathbf{M}x \rightarrow \mathbf{K}[\mathbf{m}(x)]\}$	

<sup>3</sup> We accept the ambiguity between the use of 'junction' to refer to the operator, and to refer to the result of applying the operator; this is consonant with the same ambiguity in the word 'conjunction'.

<sup>4</sup> In other words, an NP headed by a relative pronoun, such as 'who' and 'whose mother'.

2. Jay respects every woman

Jay	[+1]	respects	every	woman	[+2]
J	$\lambda x\{x_1\}$		$\lambda P_0 \wedge \{y \mid Py\}$	$\mathbf{W}_0$	
			$\wedge \{y \mid \mathbf{W}y\}$		$\lambda x\{x_2\}$
		$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\wedge \{y_2 \mid \mathbf{W}y\}$		
$J_1$			$\wedge \{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$		
			$\wedge \{ \mathbf{R}Jy \mid \mathbf{W}y \}$		
			$\forall y \{ \mathbf{W}y \rightarrow \mathbf{R}Jy \}$		

3. Jay does not respect every woman

Jay	[+1]	does-not	respect	every	woman	[+2]
J	$\lambda x\{x_1\}$			$\lambda P_0 \wedge \{y \mid Py\}$	$\mathbf{W}_0$	
				$\wedge \{y \mid \mathbf{W}y\}$		$\lambda x\{x_2\}$
			$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\wedge \{y_2 \mid \mathbf{W}y\}$		
				$\wedge \{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$		
		$\lambda P_1 \lambda x_1 \sim Px$		$\lambda x_1 \forall y \{ \mathbf{W}y \rightarrow \mathbf{R}xy \}$		
$J_1$				$\lambda x_1 \sim \forall y \{ \mathbf{W}y \rightarrow \mathbf{R}xy \}$		
				$\sim \forall y \{ \mathbf{W}y \rightarrow \mathbf{R}Jy \}$		

4. every man respects every woman

every man <sub>1</sub>	respects	every woman <sub>1</sub>
	$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\wedge \{y_2 \mid \mathbf{W}y\}$
$\wedge \{x_1 \mid \mathbf{M}x\}$	$\wedge \{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$	
$\wedge \{ \wedge \{ \mathbf{R}xy \mid \mathbf{W}y \} \mid \mathbf{M}x \}$		
$\forall x \{ \mathbf{M}x \rightarrow \forall y \{ \mathbf{W}y \rightarrow \mathbf{R}xy \} \}$		

every man <sub>1</sub>	respects	every woman <sub>2</sub>
	$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\wedge \{y_2 \mid \mathbf{W}y\}$
$\wedge \{x_1 \mid \mathbf{M}x\}$	$\wedge \{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$	
$\wedge \{ \wedge \{ \mathbf{R}xy \mid \mathbf{M}x \} \mid \mathbf{W}y \}$		
$\forall y \{ \mathbf{W}y \rightarrow \forall x \{ \mathbf{M}x \rightarrow \mathbf{R}xy \} \}$		

5. Jay does not respect some woman

Jay	[+1]	does-not	respect	some	woman	[+2]
J	$\lambda x\{x_1\}$			$\lambda P_0 \vee \{y \mid Py\}$	$\mathbf{W}_0$	
				$\vee \{y \mid \mathbf{W}y\}$		$\lambda x\{x_2\}$
			$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\vee \{y_2 \mid \mathbf{W}y\}$		
		$\lambda P_1 \lambda x_1 \{ \sim Px \}$		$\vee \{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$		
$J_1$				$\vee \{ \lambda x_1 \sim \mathbf{R}xy \mid \mathbf{W}y \}$		
				$\vee \{ \sim \mathbf{R}Jy \mid \mathbf{W}y \}$		
				$\exists y \{ \mathbf{W}y \ \& \ \sim \mathbf{R}xy \}$		

6. Jay does not respect some woman [alternative reading?]

Jay	[+1]	does-not	respect	some	woman	[+2]
J	$\lambda x\{x_1\}$			$\lambda P_0 \vee \{y \mid Py\}$	$\mathbf{W}_0$	
				$\vee \{y \mid Wy\}$		$\lambda x\{x_2\}$
			$\lambda y_2 \lambda x_1 \mathbf{R}xy$		$\vee \{y_2 \mid Wy\}$	
				$\vee \{ \lambda x_1 \mathbf{R}xy \mid Wy \}$		
		$\lambda P_1 \lambda x_1 \{ \sim Px \}$		$\lambda x_1 \exists y \{ Wy \ \& \ Rxy \}$		
J <sub>1</sub>			$\lambda x_1 \sim \exists y \{ Wy \ \& \ Rxy \}$			
$\sim \exists y \{ Wy \ \& \ Rly \}$						

7. every man respects some woman

‘every man’ has wide scope

‘some woman’ has wide scope

every man <sub>1</sub>	respects	some woman <sub>2</sub>
	$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\vee \{ y_2 \mid Wy \}$
$\wedge \{ x_1 \mid Mx \}$	$\vee \{ \lambda x_1 \mathbf{R}xy \mid Wy \}$	
$\wedge \{ \vee \{ \mathbf{R}xy \mid Wy \} \mid Mx \}$		
$\forall x \{ Mx \rightarrow \exists y \{ Wy \ \& \ Rxy \} \}$		

every man <sub>1</sub>	respects	some woman <sub>2</sub>
	$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\vee \{ y_2 \mid Wy \}$
$\wedge \{ x_1 \mid Mx \}$	$\vee \{ \lambda x_1 \mathbf{R}xy \mid Wy \}$	
$\vee \{ \wedge \{ \mathbf{R}xy \mid Mx \} \mid Wy \}$		
$\exists y \{ Wy \ \& \ \forall x \{ Mx \rightarrow \mathbf{R}xy \} \}$		

8. no man is virtuous

no	man	[+1]	is	virtuous
$\lambda P_0 \wedge \{ \lambda Q \sim Qx \mid Px \}$	$\mathbf{M}_0$		$\lambda P_0 \{ P_1 \}$	$\mathbf{V}_0$
$\wedge \{ \lambda Q \sim Qx \mid Mx \}$		$\lambda x\{x_1\}$		
$\wedge \{ \lambda Q_1 \sim Qx \mid Mx \}$		$\mathbf{V}_1$		
$\wedge \{ \sim Vx \mid Mx \}$				
$\forall x \{ Mx \rightarrow \sim Vx \}$ $\sim \exists x \{ Mx \ \& \ Vx \}$				

9. no man respects every woman

no	man	[+1]	respects	every	woman	[+2]
$\lambda P_0 \wedge \{ \lambda Q \sim Qx \mid Px \}$	$\mathbf{M}_0$			$\lambda P_0 \wedge \{ y \mid Py \}$	$\mathbf{W}_0$	
$\wedge \{ \lambda Q \sim Qx \mid Mx \}$		$\lambda x\{x_1\}$		$\wedge \{ y \mid Wy \}$		$\lambda x\{x_2\}$
			$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\wedge \{ y_2 \mid Wy \}$		
			$\wedge \{ \lambda x_1 \mathbf{R}xy \mid Wy \}$			
$\wedge \{ \lambda Q_1 \sim Qx \mid Mx \}$			$\lambda x_1 \forall y \{ Wy \rightarrow \mathbf{R}xy \}$			
$\wedge \{ \sim \forall y \{ Wy \rightarrow \mathbf{R}xy \} \mid Mx \}$						
$\forall x \{ Mx \rightarrow \sim \forall y \{ Wy \rightarrow \mathbf{R}xy \} \}$						

## 8. ‘Any’ [Sub-Junction]

In the earlier examples,

Jay does not respect **every** woman  
no man respects **every** woman

‘every’ cannot gain scope over ‘does not’ or ‘no’. If we want to express these ideas in English, we must instead use the following sentences.

Jay does not respect **any** woman  
no man respects **any** woman

The natural hypothesis is that ‘any’ is how we pronounce ‘every’ when we want it to slip outside another operator. Unfortunately, this hypothesis is defeated by the following example.

no man respects **any** woman who does not respect **him**

The problem is that ‘no man’ has to out-scope ‘any woman...him’ in order to *bind* ‘him’, so ‘any woman...him’ cannot simply be moved past ‘no man’ by quantifier-movement. We will deal with this problem in detail after we reconsider pronoun-binding.

We next observe that the word ‘any’ is quite eccentric grammatically, in a way similar to ‘ever’ and ‘either’; in particular, they are all *assertorically-deficient*. For example, if I ask

- ☺ does **anyone** have a question?
- ☺ have you **ever** been to Paris?
- ☺ do you know **either** of these people?

you are not grammatically-permitted to answer:

- ☹ yes, **anyone** has a question
- ☹ yes, I have **ever** been to Paris
- ☹ yes, I know **either** of these people

Also, one can say the following.

- ☺ **every** student is sitting

but not the following.

- ☹ **any** student is sitting

On the other hand, one can say **either** of the following.

- ☺ **every** student caught cheating will be expelled
- ☺ **any** student caught cheating will be expelled

In the last case, the difference seems to be that the latter, but not the former, carries *modal (subjunctive) force*. This explains why the following is good or bad, respectively, according to whether it is subjunctive or indicative in character.

**any** pet of mine is neutered<sup>5</sup>

---

<sup>5</sup> Actually, we only neuter our *mammalian* pets; the others remain intact!

By way of accounting for the behavior of ‘any’, we propose the following as our overall hypothesis.

‘any A is B’ is not independently-assertive;  
rather, it is **sub-assertional**.

In order to formalize this hypothesis, we propose yet another junction,  $\Pi$ ,<sup>6</sup> called *sub-junction*, with its own special properties, which are summarized as follows.

- (1) if  $\mathfrak{S}$  is a type, then so is  $\Pi\mathfrak{S}$
- (2)  $\Pi\mathfrak{S} \neq \mathfrak{S}$
- (3)  $\Pi$  *never* simplifies [sub-assertional]
- (4)  $\Pi$  admits all expressions, some of which promote  $\Pi$  to  $\wedge$  ( $\Pi$ -promotion)

## 1. Examples

1. Jay does not respect any woman

Jay	[+1]	does-not	respect	any	woman	[+2]
J	$\lambda x\{x_1\}$			$\lambda P_0\Pi\{y \mid Py\}$	$\mathbf{W}_0$	
				$\Pi\{y \mid \mathbf{W}y\}$		$\lambda x\{x_2\}$
			$\lambda y_2\lambda x_1\mathbf{R}xy$		$\Pi\{y_2 \mid \mathbf{W}y\}$	
		$\lambda P_1\lambda x_1\{\sim Px\}$		$\Pi\{\lambda x_1\mathbf{R}xy \mid \mathbf{W}y\}$		
$J_1$			$\wedge\{\lambda x_1\sim\mathbf{R}xy \mid \mathbf{W}y\}$			①
			$\wedge\{\sim\mathbf{R}ly \mid \mathbf{W}y\}$			
			$\forall y\{\mathbf{W}y \rightarrow \sim\mathbf{R}xy\}$			

Note, in particular, that (unlike  $\wedge$ )  $\Pi$  admits  $\llbracket$ does not $\rrbracket$ , which moreover promotes it to  $\wedge$ , which enables us to treat the resulting phrase as genuinely assertive.

2. no man respects any woman

no	man	[+1]	respects	any	woman	[+2]
$\lambda P_0\wedge\{\lambda Q\sim Qx \mid Px\}$	$\mathbf{M}_0$			$\lambda P_0\Pi\{y \mid Py\}$	$\mathbf{W}_0$	
$\wedge\{\lambda Q\sim Qx \mid \mathbf{M}x\}$		$\lambda x\{x_1\}$		$\Pi\{y \mid \mathbf{W}y\}$		$\lambda x\{x_2\}$
			$\lambda y_2\lambda x_1\mathbf{R}xy$		$\Pi\{y_2 \mid \mathbf{W}y\}$	
		$\wedge\{\lambda Q_1\sim Qx \mid \mathbf{M}x\}$		$\Pi\{\lambda x_1\mathbf{R}xy \mid \mathbf{W}y\}$		
			$\wedge\{\lambda Q_1\sim Qx \times \Pi\{\lambda x_1\mathbf{R}xy \mid \mathbf{W}y\} \mid \mathbf{M}x\}$			①
			$\wedge\{\wedge\{\sim\mathbf{R}xy \mid \mathbf{W}y\} \mid \mathbf{M}x\}$			②
			$\forall x\{\mathbf{M}x \rightarrow \forall y\{\mathbf{W}y \rightarrow \sim\mathbf{R}xy\}\}$			③

<sup>6</sup> Cyrillic letter ‘el’, which is short for ‘любой’ [‘liuboi’], which is Russian for (approximately) ‘any one’.

This derivation treats ‘no man’ as having wide scope. This is accomplished on line ① by the  $\wedge$ -expression absorbing the  $\Pi$ -expression in the absolutely minimal way, yielding the shaded material. Then ② is obtained from ① by applying junction-combination to the shaded material, noting that  $\llbracket$ does not $\rrbracket$  promotes  $\Pi$  to  $\wedge$ . Finally, ③ is obtained from ② by two applications of  $\wedge$ -simplification.

The following is an alternative derivation, which grants ‘any woman’ wide-scope.

no	man	[+1]	respects	any	woman	[+2]	
$\lambda P_0 \wedge \{ \lambda Q \sim Qx \mid Px \}$	$\mathbf{M}_0$			$\lambda P_0 \Pi \{ y \mid Py \}$	$\mathbf{W}_0$		
$\wedge \{ \lambda Q \sim Qx \mid \mathbf{M}x \}$		$\lambda x \{ x_1 \}$		$\Pi \{ y \mid \mathbf{W}y \}$		$\lambda x \{ x_2 \}$	
			$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\Pi \{ y_2 \mid \mathbf{W}y \}$			
$\wedge \{ \lambda Q_1 \sim Qx \mid \mathbf{M}x \}$			$\Pi \{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$				
$\wedge \{ \wedge \{ \lambda Q_1 \sim Qx \mid \mathbf{M}x \} \times \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$							①
$\wedge \{ \wedge \{ \sim \mathbf{R}xy \mid \mathbf{M}x \} \mid \mathbf{W}y \}$							②
$\forall y \{ \mathbf{W}y \rightarrow \forall x \{ \mathbf{M}x \rightarrow \sim \mathbf{R}xy \} \}$							③

In this derivation, by way of constructing ①, the  $\Pi$ -expression absorbs the  $\wedge$ -expression, which promotes  $\Pi$  to  $\wedge$ . Notice that the two readings are logically equivalent.

So far, we have noted in passing two examples of  $\Pi$ -promotion, which we now officially acknowledge.

( $\Pi_1$ )  $\llbracket$ does-not $\rrbracket$  promotes  $\Pi$  to  $\wedge$ .

( $\Pi_2$ )  $\llbracket$ no P $\rrbracket$  promotes  $\Pi$  to  $\wedge$ .

To these, we now add the following,

( $\Pi_3$ )  $\llbracket$ if $\rrbracket$  promotes  $\Pi$  to  $\wedge$ .

which is illustrated in the following example.

3. if Jay respects any woman, Jay respects Kay

if	Jay <sub>1</sub>	respects	any	woman	[+2]	Jay respects Kay
			$\lambda P_0 \Pi \{ y \mid Py \}$	$\mathbf{W}_0$		
			$\Pi \{ y \mid \mathbf{W}y \}$		$\lambda x \{ x_2 \}$	
		$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\Pi \{ y_2 \mid \mathbf{W}y \}$			
	J <sub>1</sub>	$\Pi \{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$				
$\lambda P \lambda Q \{ P \rightarrow Q \}$	$\Pi \{ \mathbf{R}Jy \mid \mathbf{W}y \}$					
①	$\wedge \{ \lambda Q \{ \mathbf{R}Jy \rightarrow Q \} \mid \mathbf{W}y \}$					$\mathbf{R}JK$
$\wedge \{ \mathbf{R}Jy \rightarrow \mathbf{R}JK \mid \mathbf{W}y \}$						
$\forall x \{ \mathbf{W}x \rightarrow \{ \mathbf{R}Jx \rightarrow \mathbf{R}JK \} \}$						

4. if Jay respects every woman, Jay respects Kay

if	Jay <sub>1</sub>	respects	every	woman	[+2]	Jay respects Kay	
$\lambda P \lambda Q \{ P \rightarrow Q \}$	J <sub>1</sub>		$\lambda P_0 \wedge \{ y   P y \}$	<b>W<sub>0</sub></b>	$\lambda x \{ x_2 \}$		
			$\wedge \{ y   W y \}$				
			$\lambda y_2 \lambda x_1 R x y$	$\wedge \{ y_2   W y \}$			
			$\wedge \{ \lambda x_1 R x y   W y \}$				
			$\wedge \{ R j y   W y \}$ $\forall y \{ W y \rightarrow R j y \}$				
$\lambda Q \{ \forall y \{ W y \rightarrow R j y \} \rightarrow Q \}$					<b>R<sub>JK</sub></b>	①	
$\forall y \{ W y \rightarrow R j y \} \rightarrow R_{JK}$							

Note: item ① involves a type-logical transformation, which is required since we have the following further admissibility restriction.

( $\wedge_4$ )  $\wedge$  does not admit  $\lambda P \lambda Q \{ P \rightarrow Q \}$  ('if').

5. no man recommends any woman to any man

no man <sub>1</sub>	recommends	any woman <sub>2</sub>	to any man	
$\wedge \{ \lambda P_1 \sim P x   M x \}$	$\lambda y_2 \lambda z_3 \lambda x_1 R x y z$	$\Pi \{ y_2   W y \}$	$\Pi \{ z_3   M z \}$	
	$\Pi \{ \lambda z_3 \lambda x_1 R x y z   W y \}$			
	$\Pi \{ \Pi \{ \lambda x_1 R x y z   M z \}   W y \}$			
	$\wedge \{ \lambda P_1 \sim P x \times \Pi \{ \Pi \{ \lambda x_1 R x y z   M z \}   W y \}   M x \}$			
	$\wedge \{ \wedge \{ \lambda P_1 \sim P x \times \Pi \{ \lambda x_1 R x y z   M z \}   W y \}   M x \}$			
$\wedge \{ \wedge \{ \wedge \{ \lambda P_1 \sim P x \times \lambda x_1 R x y z   M z \}   W y \}   M x \}$				
$\wedge \{ \wedge \{ \wedge \{ \sim R x y z   M z \}   W y \}   M x \}$				
$\forall x \{ M x \rightarrow \forall y \{ W y \rightarrow \forall z \{ M z \rightarrow \sim R x y z \} \}$				

9. Appendix – The Cyrillic Letter ‘Л’ Sometimes Looks Like ‘Λ’

