1. New Theory of Quantifiers

We propose an account of quantifiers in terms of ***junctions***, which are infinitary-operators.[1] All told (or perhaps, all *tolled*), we will propose six different junctions, as follows.

⋀	conjunction	every
⋁	disjunction	some
Л	subjunction	any
⊠	exclusive-disjunction	exactly-one
Σ	sum	"a" (base)[2]
Π	product	"a" (promoted)

2. General Formation Rules for Junctions

First, for each junction Ж, we posit a new type-forming operator Ж, defined so that

if $\mathfrak{I}$ is a type, then so is Ж$\mathfrak{I}$.

We further propose the following formation rule.

If ν is a variable, ε is an expression of type $\mathfrak{I}$, and Φ is a formula, then

$Ж_{\nu}\{\varepsilon \mid \Phi\}$

is an expression of type Ж$\mathfrak{I}$, which we read as:

the Ж (over ν) of all ε such that Φ

where 'Ж' is replaced by the appropriate term (e.g., 'conjunction' or 'product').

If ε and Φ share exactly-one free variable, then this is understood to be the variable of abstraction, so we omit it, in which case we have (informal) expressions of the following form,

$Ж\{\varepsilon \mid \Phi\}$

which we read as:

the Ж of all ε such that Φ.

3. Interpretation of Quantifiers (so far)

⟦every⟧	=	$\lambda P_0 \bigwedge\{x \mid Px\}$
⟦some⟧	=	$\lambda P_0 \bigvee\{x \mid Px\}$
⟦no⟧	=	$\lambda P_0 \bigwedge\{\lambda Q{\sim}Qx \mid Px\}$
⟦any⟧	=	λP_0 Л$\{x \mid Px\}$

[1] By an infinitary-operator, we mean an operator that acts on *a collection of arguments* of arbitrary size, even infinite, and even empty.
[2] The word 'a' is scare-quoted because it is often not pronounced. In English, singular common-noun-phrases *require* 'a', but plural-phrases and mass-phrases *prohibit* 'a'. By comparison, Spanish has plural forms of 'a' ['unas' and 'unos'].

4. Composition (so far)

Junction-Composition (Ж-Com)	
α	α is any expression (**or null**)
$Ж_v\{\beta \mid \Phi\}$	Ж **admits** α
$\{\alpha,\beta\} \vdash \gamma$	a sub-derivation of γ from $\{\alpha,\beta\}$
$Ж^*_v\{\gamma \mid \Phi\}$	Ж* is a *transform* of Ж

Here, α, β, γ are expressions, and Ж is any junction. The nature of Ж*, and the admissibility restrictions are gradually spelled out over these pages.

5. Admissibility Restrictions (so far)

($\wedge_1$) $\wedge$ does not admit $\lambda P_1 \lambda x_1 {\sim} Px$ ['does-not'].

($\wedge_2$) $\wedge$ does not admit any relative-pronoun-phrase.[3]

($\wedge_3$) $\wedge$ does not admit $\wedge\{\ \lambda Q{\sim}Qx \mid Px\ \}$ ['no P'].

($\wedge_4$) $\wedge$ does not admit $\lambda P \lambda Q\{P \rightarrow Q\}$ ['if'].

6. Simplification Principles (so far)

$\wedge_v\{\Psi \mid \Phi\}$ / $\forall v\{\Phi \rightarrow \Psi\}$ [$\wedge$-simplification]

$\vee_v\{\Psi \mid \Phi\}$ / $\exists v\{\Phi \& \Psi\}$ [$\vee$-simplification]

7. Transform Principles (so far)

(Л$_1$) anti-tonic functions transform (promote) Л to $\wedge$.

8. Examples

1. every man respects every woman

every man$_1$	respects	every woman$_1$
	$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\wedge\{y_2 \mid \mathbf{W}y\}$
$\wedge\{x_1 \mid \mathbf{M}x\}$	$\wedge\{\ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\ \}$	
$\wedge\{\ \wedge\{\ \mathbf{R}xy \mid \mathbf{W}y\ \} \mid \mathbf{M}x\ \}$		
$\forall x\ \{\ \mathbf{M}x \rightarrow \forall y\ \{\mathbf{W}y \rightarrow \mathbf{R}xy\}\ \}$		

every man$_1$	respects	every woman$_2$
	$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\wedge\{y_2 \mid \mathbf{W}y\}$
$\wedge\{x_1 \mid \mathbf{M}x\}$	$\wedge\{\ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\ \}$	
$\wedge\{\ \wedge\{\ \mathbf{R}xy \mid \mathbf{M}x\ \} \mid \mathbf{W}y\ \}$		
$\forall y\ \{\ \mathbf{W}y \rightarrow \forall x\ \{\mathbf{M}x \rightarrow \mathbf{R}xy\}\ \}$		

[3] In other words, an NP headed by a relative pronoun, such as 'who' and 'whose mother'.

2. Jay does not respect some woman

<table>
<tr><td>Jay $_1$</td><td>does-not</td><td>respect</td><td>some woman $_2$</td></tr>
<tr><td rowspan="3">J_1</td><td rowspan="2">$\lambda P_1 \lambda x_1\{\sim Px\}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$\vee\{y_2 \mid \mathbf{W}y\}$</td></tr>
<tr><td colspan="2">$\vee\{\ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="3">$\vee\{\ \lambda x_1 \sim \mathbf{R}xy \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="4">$\vee\{\ \sim \mathbf{R}Jy \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="4">$\exists y\ \{\ \mathbf{W}y\ \&\ \sim \mathbf{R}xy\ \}$</td></tr>
</table>

<table>
<tr><td>Jay $_1$</td><td>does-not</td><td>respect</td><td>some woman $_2$</td></tr>
<tr><td rowspan="4">J_1</td><td rowspan="3">$\lambda P_1 \lambda x_1\{\sim Px\}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$\vee\{y_2 \mid \mathbf{W}y\}$</td></tr>
<tr><td colspan="2">$\vee\{\ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="2">$\lambda x_1\ \exists y\{\mathbf{W}y\ \&\ \mathbf{R}xy\}$</td></tr>
<tr><td colspan="3">$\lambda x_1\ \sim\exists y\{\mathbf{W}y\ \&\ \mathbf{R}xy\}$</td></tr>
<tr><td colspan="4">$\sim\exists y\ \{\ \mathbf{W}y\ \&\ \mathbf{R}Jy\ \}$</td></tr>
</table>

3. every man respects some woman

<table>
<tr><td>every man $_1$</td><td>respects</td><td>some woman $_2$</td></tr>
<tr><td rowspan="2">$\wedge\{\ x_1 \mid \mathbf{M}x\ \}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$\vee\{\ y_2 \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="2">$\vee\{\ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="3">$\wedge\{\ \vee\{\ \mathbf{R}xy \mid \mathbf{W}y\ \} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="3">$\forall x\ \{\ \mathbf{M}x \rightarrow \exists y\{\mathbf{W}y\ \&\ \mathbf{R}xy\}\ \}$</td></tr>
</table>

<table>
<tr><td>every man $_1$</td><td>respects</td><td>some woman $_2$</td></tr>
<tr><td rowspan="2">$\wedge\{\ x_1 \mid \mathbf{M}x\ \}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$\vee\{\ y_2 \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="2">$\vee\{\ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="3">$\vee\{\ \wedge\{\ \mathbf{R}xy \mid \mathbf{M}x\ \} \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="3">$\exists y\{\ \mathbf{W}y\ \&\ \forall x\ \{\mathbf{M}x \rightarrow \mathbf{R}xy\}\ \}$</td></tr>
</table>

4. no man respects every woman

<table>
<tr><td>no</td><td>man</td><td>[+1]</td><td>respects</td><td>every</td><td>woman</td><td>[+2]</td></tr>
<tr><td>$\lambda P_0 \wedge\{\lambda Q \sim Qx \mid Px\}$</td><td>$\mathbf{M}_0$</td><td rowspan="2">$\lambda x\{x_1\}$</td><td rowspan="3">$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$\lambda P_0 \wedge\{y \mid Py\}$</td><td>$\mathbf{W}_0$</td><td rowspan="2">$\lambda x\{x_2\}$</td></tr>
<tr><td colspan="2">$\wedge\{\ \lambda Q \sim Qx \mid \mathbf{M}x\ \}$</td><td colspan="2">$\wedge\{\ y \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="3" rowspan="3">$\wedge\{\ \lambda Q_1 \sim Qx \mid \mathbf{M}x\ \}$</td><td colspan="3">$\wedge\{\ y_2 \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="4">$\wedge\{\ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="4">$\lambda x_1\ \forall y\{\mathbf{W}y \rightarrow \mathbf{R}xy\}$</td></tr>
<tr><td colspan="7">$\wedge\{\ \sim\forall y\{\mathbf{W}y \rightarrow \mathbf{R}xy\} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="7">$\forall x\ \{\ \mathbf{M}x \rightarrow \sim\forall y\{\mathbf{W}y \rightarrow \mathbf{R}xy\}\ \}$</td></tr>
</table>

5. Jay does not respect any woman

<table>
<tr><td>Jay</td><td>[+1]</td><td>does-not</td><td>respect</td><td>any</td><td>woman</td><td>[+2]</td></tr>
<tr><td>J</td><td>$\lambda x\{x_1\}$</td><td rowspan="4">$\lambda P_1 \lambda x_1\{\sim Px\}$</td><td rowspan="3">$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>$\lambda P_0 \text{Л}\{y \mid Py\}$</td><td>$\mathbf{W}_0$</td><td rowspan="2">$\lambda x\{x_2\}$</td></tr>
<tr><td colspan="2" rowspan="4">J_1</td><td colspan="2">$\text{Л}\{\ y \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="3">$\text{Л}\{\ y_2 \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="4">$\text{Л}\{\ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="5">$\wedge\{\ \lambda x_1 \sim \mathbf{R}xy \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="7">$\wedge\{\ \sim \mathbf{R}Jy \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="7">$\forall y\ \{\ \mathbf{W}y \rightarrow \sim \mathbf{R}xy\ \}$</td></tr>
</table>

6. no man respects any woman

no	man	[+1]	respects	any	woman	[+2]
$\lambda P_0 \wedge \{\lambda Q \sim Qx \mid Px\}$	$\mathbf{M}_0$			$\lambda P_0 \text{Л}\{y \mid Py\}$	$\mathbf{W}_0$	
$\wedge\{ \lambda Q \sim Qx \mid \mathbf{M}x \}$		$\lambda x\{x_1\}$		$\text{Л}\{ y \mid \mathbf{W}y \}$		$\lambda x\{x_2\}$
			$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\text{Л}\{ y_2 \mid \mathbf{W}y \}$		
$\wedge\{ \lambda Q_1 \sim Qx \mid \mathbf{M}x \}$			$\text{Л}\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$			
$\wedge\{ \lambda Q_1 \sim Qx \times \text{Л}\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \} \mid \mathbf{M}x \}$						
$\wedge\{ \wedge\{ \sim \mathbf{R}xy \mid \mathbf{W}y \} \mid \mathbf{M}x \}$						
$\forall x \{ \mathbf{M}x \rightarrow \forall y\{\mathbf{W}y \rightarrow \sim \mathbf{R}xy\} \}$						

no	man	[+1]	respects	any	woman	[+2]
$\lambda P_0 \wedge \{\lambda Q \sim Qx \mid Px\}$	$\mathbf{M}_0$			$\lambda P_0 \text{Л}\{y \mid Py\}$	$\mathbf{W}_0$	
$\wedge\{ \lambda Q \sim Qx \mid \mathbf{M}x \}$		$\lambda x\{x_1\}$		$\text{Л}\{ y \mid \mathbf{W}y \}$		$\lambda x\{x_2\}$
			$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\text{Л}\{ y_2 \mid \mathbf{W}y \}$		
$\wedge\{ \lambda Q_1 \sim Qx \mid \mathbf{M}x \}$			$\text{Л}\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$			
$\wedge\{ \wedge\{ \lambda Q_1 \sim Qx \mid \mathbf{M}x \} \times \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$						
$\wedge\{ \wedge\{ \sim \mathbf{R}xy \mid \mathbf{M}x \} \mid \mathbf{W}y \}$						
$\forall y \{ \mathbf{W}y \rightarrow \forall x\{\mathbf{M}x \rightarrow \sim \mathbf{R}xy\} \}$						

7. if Jay respects any woman, Jay respects Kay

if	Jay$_1$	respects	any	woman	[+2]	Jay respects Kay
			$\lambda P_0 \text{Л}\{ y \mid Py \}$	$\mathbf{W}_0$		
			$\text{Л}\{ y \mid \mathbf{W}y \}$		$\lambda x\{x_2\}$	
		$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\text{Л}\{ y_2 \mid \mathbf{W}y \}$			
	J_1	$\text{Л}\{ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y \}$				
$\lambda P \lambda Q\{P \rightarrow Q\}$	$\text{Л}\{ \mathbf{R}\text{J}y \mid \mathbf{W}y \}$					
$\wedge\{ \lambda Q\{\mathbf{R}\text{J}y \rightarrow Q\} \mid \mathbf{W}y \}$						$\mathbf{R}\text{JK}$
$\wedge\{ \mathbf{R}\text{J}y \rightarrow \mathbf{R}\text{JK} \mid \mathbf{W}y \}$						
$\forall x \{ \mathbf{W}x \rightarrow \{\mathbf{R}\text{J}x \rightarrow \mathbf{R}\text{JK}\} \}$						

8. if Jay respects every woman, Jay respects Kay

<table>
<tr><td>if</td><td>Jay$_1$</td><td>respects</td><td>every</td><td>woman</td><td>[+2]</td><td>Jay respects Kay</td></tr>
<tr><td></td><td></td><td></td><td>$\lambda P_0 \wedge\{y \mid Py\}$</td><td>$\mathbf{W}_0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td colspan="2">$\wedge\{\ y \mid \mathbf{W}y\ \}$</td><td>$\lambda x\{x_2\}$</td><td></td></tr>
<tr><td></td><td></td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td colspan="3">$\wedge\{\ y_2 \mid \mathbf{W}y\ \}$</td><td></td></tr>
<tr><td></td><td>J_1</td><td colspan="4">$\wedge\{\ \lambda x_1 \mathbf{R}xy \mid \mathbf{W}y\ \}$</td><td></td></tr>
<tr><td>$\lambda P \lambda Q\{P \rightarrow Q\}$</td><td colspan="5">$\wedge\{\ \mathbf{R}\text{J}y \mid \mathbf{W}y\ \}$
$\forall y\{\mathbf{W}y \rightarrow \mathbf{R}\text{J}y\}$</td><td></td></tr>
<tr><td colspan="6">$\lambda Q\ \{\ \forall y\{\mathbf{W}y \rightarrow \mathbf{R}\text{J}y\} \rightarrow Q\ \}$</td><td>$\mathbf{R}\text{JK}$</td></tr>
<tr><td colspan="7">$\forall y\{\mathbf{W}y \rightarrow \mathbf{R}\text{J}y\} \rightarrow \mathbf{R}\text{JK}$</td></tr>
</table>

9. no man recommends any woman to any man

<table>
<tr><td>no man$_1$</td><td>recommends</td><td>any woman$_2$</td><td>to any man</td></tr>
<tr><td></td><td>$\lambda y_2 \lambda z_3 \lambda x_1 \mathbf{R}xyz$</td><td>$\text{Л}\{y_2 \mid \mathbf{W}y\}$</td><td></td></tr>
<tr><td></td><td colspan="2">$\text{Л}\{\ \lambda z_3 \lambda x_1 \mathbf{R}xyz \mid \mathbf{W}y\ \}$</td><td>$\text{Л}\{z_3 \mid \mathbf{M}z\}$</td></tr>
<tr><td>$\wedge\{\ \lambda P_1 {\sim} Px \mid \mathbf{M}x\ \}$</td><td colspan="3">$\text{Л}\{\ \text{Л}\{\ \lambda x_1 \mathbf{R}xyz \mid \mathbf{M}z\ \} \mid \mathbf{W}y\ \}$</td></tr>
<tr><td colspan="4">$\wedge\{\ \lambda P_1 {\sim} Px \times \text{Л}\{\ \text{Л}\{\ \lambda x_1 \mathbf{R}xyz \mid \mathbf{M}z\ \} \mid \mathbf{W}y\ \} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="4">$\wedge\{\ \wedge\{\ \lambda P_1 {\sim} Px \times\ \text{Л}\{\ \lambda x_1 \mathbf{R}xyz \mid \mathbf{M}z\ \} \mid \mathbf{W}y\ \} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="4">$\wedge\{\ \wedge\{\ \wedge\{\ \lambda P_1 {\sim} Px \times \lambda x_1 \mathbf{R}xyz \mid \mathbf{M}z\ \} \mid \mathbf{W}y\ \} \mid \mathbf{M}x\ \}$</td></tr>
<tr><td colspan="4">$\wedge\big\{\ \wedge\{\ \wedge\{\ {\sim}\mathbf{R}xyz \mid \mathbf{M}z\ \} \mid \mathbf{W}y\ \} \mid \mathbf{M}\ x\ \big\}$</td></tr>
<tr><td colspan="4">$\forall x\big\{\ \mathbf{M}x \rightarrow \forall y\{\ \mathbf{W}y \rightarrow \forall z\{\mathbf{M}z \rightarrow {\sim}\mathbf{R}xyz\}\ \}\ \big\}$</td></tr>
</table>

9. Pronoun-Binding

We now return to pronoun-binding, problems with which set us on this path in the first place. We follow the same basic binding principles as before. In particular, we propose two anaphoric morphemes

(α) creates an anaphoric-role (where α is a negative integer)
[α] produces an item to fill (bind) an anaphoric role created by (α)

which are type-logically rendered as follows.[4]

$$[\![\ (\alpha)\ ]\!] =_{\text{df}} \lambda x_\alpha\{x\}$$
$$[\![\ [\alpha]\ ]\!] =_{\text{df}} \lambda x\{x \times x_\alpha\}$$

[4] The type-logical rendering of [α] will be generalized considerably for junctions.

We also propose that an essentially-anaphoric pronoun ‘e’ is semantically vacuous. Rather, such an expression serves as merely a vehicle/anchor for attaching inflections, including theta-markers and alpha-markers.[5]

The following are simple examples of how binding works.

1. Jay respects his mother

Jay	[–1]	[+1]	respects	(–1)	his		mother	[+2]
					he	's		
J	$\lambda x\{x \times x_{-1}\}$			$\lambda x_{-1}\{x\}$	$\varnothing$			
$\text{J} \times \text{J}_{-1}$		$\lambda x\{x_1\}$		$\lambda x_{-1}\{x\}$		$\lambda x\{x_6\}$		
				$\lambda x_{-1}\{x_6\}$			$\lambda x_6\{\mathbf{m}(x)\}$	
				$\lambda x_{-1}\{\mathbf{m}(x)\}$				$\lambda x\{x_2\}$
			$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\lambda x_{-1}\{\mathbf{m}(x_2)\}$				
$\text{J}_1 \times \text{J}_{-1}$			$\lambda y_{-1}\ \lambda x_1\ \mathbf{R}[x,\mathbf{m}(y)]$					
$\mathbf{R}[\text{J},\mathbf{m}(\text{J})]$								

2. Kay's father respects her mother

Kay	[–1]	's	father	[+1]	respects	(–1)	her		mother	[+2]
							e+F	[+6]		
K	$\lambda x\{x \times x_{-1}\}$					$\lambda x_{-1}\{x\}$	$\varnothing$	$\lambda x\{x_6\}$		
$\text{K} \times \text{K}_{-1}$		$\lambda x\{x_6\}$				$\lambda x_{-1}\{x_6\}$			$\lambda x_6\{\mathbf{m}(x)\}$	
$\text{K}_6 \times \text{K}_{-1}$			$\lambda x_6\{\mathbf{f}(x)$			$\lambda x_{-1}\{\mathbf{m}(x)\}$				$\lambda x\{x_2\}$
$\mathbf{f}(\text{K}) \times \text{K}_{-1}$				$\lambda x\{x_1\}$	$\lambda y_2 \lambda x_1 \mathbf{R}xy$	$\lambda x_{-1}\{\mathbf{m}(x)_2\}$				
$\mathbf{f}(\text{K})_1 \times \text{K}_{-1}$					$\lambda y_{-1}\ \lambda x_1\ \mathbf{R}[x,\ \mathbf{m}(y)]$					
$\mathbf{f}(\text{K})_1 \times \lambda x_1\ \mathbf{R}[x,\ \mathbf{m}(\text{K})]$										
$\mathbf{R}[\ \mathbf{f}(\text{K}),\ \mathbf{m}(\text{K})\ ]$										

10. Junctions and Pronoun-Binding

There are numerous problems with our theory of pronoun-binding when it applies to quantifier phrases, as they are originally formulated in terms of second-order predicates. Let us now see how binding works when applied to junctions.

[5] It also serves to anchor gender-markers and number-markers, which we ignore for the moment in the computations.

1. every man respects his mother

<table>
<tr><td>every</td><td>man</td><td>[–1]</td><td>[+1]</td><td>respects (–1) his mother [+2]</td></tr>
<tr><td>$\lambda P_0 \wedge \{x \mid Px\}$</td><td>$\mathbf{M}_0$</td><td rowspan="2">$\lambda x\{x \times x_{-1}\}$</td><td rowspan="3">$\lambda x\{x_1\}$</td><td rowspan="4">$\lambda y_{-1}\, \lambda x_1\, \mathbf{R}[x,\mathbf{m}(y)]$</td></tr>
<tr><td colspan="2">$\wedge\{\, x \mid \mathbf{M}x \,\}$</td></tr>
<tr><td colspan="3">$\wedge\{\, x \times x_{-1} \mid \mathbf{M}x \,\}$</td></tr>
<tr><td colspan="4">$\wedge\{\, x_1 \times x_{-1} \mid \mathbf{M}x \,\}$</td></tr>
<tr><td colspan="5">$\wedge\{x_1 \times x_{-1} \times \lambda y_{-1}\, \lambda x_1\, \mathbf{R}[x,\mathbf{m}(y)] \mid \mathbf{M}x\}$</td></tr>
<tr><td colspan="5">$\wedge\{x_1 \times \lambda x_1\, \mathbf{R}[x,\mathbf{m}(x)] \mid \mathbf{M}x\}$</td></tr>
<tr><td colspan="5">$\wedge\{\, \mathbf{R}[x,\mathbf{m}(x)] \mid \mathbf{M}x\}$</td></tr>
<tr><td colspan="5">$\forall x\, \{\, \mathbf{M}x \rightarrow \mathbf{R}[x,\mathbf{m}(x)] \,\}$</td></tr>
</table>

2. every man is happy if he is virtuous

<table>
<tr><td>every</td><td>man</td><td>[–1]</td><td>[+1]</td><td>is</td><td>happy</td><td>if</td><td>(–1)</td><td>he</td><td>[+1]</td><td>is</td><td>virtuous</td></tr>
<tr><td>$\lambda P_0 \wedge \{x \mid Px\}$</td><td>$\mathbf{M}_0$</td><td rowspan="2">$\lambda x\{x \times x_{-1}\}$</td><td rowspan="3">$\lambda x\{x_1\}$</td><td colspan="2" rowspan="4">$\lambda x_1 \mathbf{H}x$</td><td rowspan="3">$\lambda P \lambda Q\{P \rightarrow Q\}$</td><td>$\lambda x_{-1}\{x\}$</td><td>$\varnothing$</td><td>$\lambda x\{x_1\}$</td><td colspan="2" rowspan="2">$\lambda x_1 \mathbf{V}x$</td></tr>
<tr><td colspan="2">$\wedge\{\, x \mid \mathbf{M}x \,\}$</td><td colspan="3">$\lambda x_{-1}\{x_1\}$</td></tr>
<tr><td colspan="3">$\wedge\{\, x \times x_{-1} \mid \mathbf{M}x \,\}$</td><td colspan="5">$\lambda x_{-1} \mathbf{V}x$</td></tr>
<tr><td colspan="4">$\wedge\{\, x_1 \times x_{-1} \mid \mathbf{M}x \,\}$</td><td colspan="6" rowspan="2">$\lambda x_{-1}\, \lambda Q\, \{\mathbf{V}x \rightarrow Q\}$</td></tr>
<tr><td colspan="6">$\wedge\{\, \mathbf{H}x \times x_{-1} \mid \mathbf{M}x \,\}$</td></tr>
<tr><td colspan="12">$\wedge\{\, \mathbf{H}x \times x_{-1} \times \lambda x_{-1}\, \lambda Q\{\mathbf{V}x \rightarrow Q\} \mid \mathbf{M}x \,\}$</td></tr>
<tr><td colspan="12">$\wedge\{\, \mathbf{H}x \times \lambda Q\{\mathbf{V}x \rightarrow Q\} \mid \mathbf{M}x \,\}$</td></tr>
<tr><td colspan="12">$\wedge\{\, \{\mathbf{V}x \rightarrow \mathbf{H}x\} \mid \mathbf{M}x \,\}$</td></tr>
<tr><td colspan="12">$\forall x\, \{\, \mathbf{M}x \rightarrow \{\mathbf{V}x \rightarrow \mathbf{H}x\} \,\}$</td></tr>
</table>

Note carefully that this is the second example of an anti-tonic function – namely ⟦if (-1)he is virtuous⟧ – that is admitted by $\wedge$; the first one is ⟦every⟧. It remains a puzzle to figure out what underlying principle can account for these, and perhaps other, exceptions to the initial principle that $\wedge$ admits no anti-tonic functions.

11. More Difficult Examples

Not only does the new theory of quantifiers make computations involving anaphora much easier, it provides an account of examples that are intractable using the old theory . For example, the earlier theory predicts the wrong semantic-value for 'no-if' sentences.

By contrast, the new theory proposes the following analysis, which is **much better**.

3. no man is happy if he is virtuous

<table>
<tr><td></td><td>no</td><td>man</td><td>[+1]</td><td>[–1]</td><td>is happy</td><td>if</td><td>(–1)</td><td>he</td><td>[+1]</td><td>is virtuous</td></tr>
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Note carefully that we propose to greatly expand our account of $[-m]$ in order to deal properly with junctions, which we do as follows, in which α is any anaphoric-marker [negative integer], and in which 'Ж' is an object-language variable ranging over junctions.

$$[\alpha] \quad \curvearrowright \quad \text{Ж}_\alpha$$
$$=_{\text{df}} \quad \lambda\ \text{Ж}_\nu\{\varepsilon \mid \Phi\}\ \text{Ж}_\nu\{\varepsilon \times \nu_\alpha \mid \Phi\}$$

This reduces to our earlier account for simple D-phrases like 'Jay' and 'Kay', based on the following theorem[6] that every D-item ∂ is equal to a junction.[7]

$$\partial \quad = \quad \wedge\{x \mid x{=}\partial\}$$

It is left as an exercise to go back and redo examples 1 and 2 using the new account of $[-m]$.

This version of anaphoric-binding allows us to apply the anaphoric-binder [–1] at later points in the derivation, as illustrated in the following alternative version of the above tree.

<table>
<tr><td></td><td>no</td><td>man</td><td>[+1]</td><td>is happy</td><td>[–1]</td><td>if</td><td>(–1)</td><td>he</td><td>[+1]</td><td>is virtuous</td></tr>
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[6] An important question is whether this theorem is provable from earlier principles. Whether it is or not, it is so plausible that our theory must contain it – either as derived, or as primitive.

[7] This identity holds for every junction except Л.