1. Indefinite Noun Phrases

In English, and many other languages, a common-noun-phrase may be prefixed by an indefinite article, the resulting phrase being what we shall call an *indefinite noun phrase*. The following are example sentences from English, in which 'a' serves as an indefinite article.

> Rex is **a** dog
> Jay owns **a** dog
> **a** dog is in the yard
> there is **a** dog in the yard
> if **a** dog is well-fed then it is happy
> every man who owns **a** dog feeds it
> if **a** man owns **a** dog, then he feeds it
> **a** dog is **a** mammal
> **a** dog can hear sounds **a** human can't
> Jay is looking for **a** dog

Note in particular that, if we delete the word 'a', we obtain phrases that standard English rejects as syntactically ill-formed.[1] On the other hand, languages that lack indefinite articles – the biggest of which are Latin, Russian, and Mandarin – have no problem saying sentences corresponding to 'that is dog'.

Furthermore, even English eschews indefinite articles when the common-nouns are plural-nouns or mass-nouns, as in the following examples.[2]

those are dogs	that is milk
Jay owns dogs	Jay has milk
dogs are in the yard	milk is in the refrigerator
there are dogs in the yard	there is milk in the refrigerator
if dogs are well-fed, they are happy	if milk is not refrigerated, it curdles
every man who owns dogs feeds them	every man who has milk drinks it
if men own dogs, they feed them	if men have milk, they drink it
dogs are mammals	milk is food
dogs can hear sounds humans can't	milk can be made into cheese
Jay is looking for dogs	Jay is looking for milk

Given the strong structural similarities between these examples, we propose to use the term 'indefinite noun phrase' in reference to all such phrases, whether prefixed by an explicit indefinite article or not.

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[1] Supposing we reject the reading according to which 'dog' is a proper-name, and the reading according to which 'dog' is a mass-noun [referring presumably to dog-matter].

[2] For the sake of comparison, Spanish and French have plural indefinite articles – 'unos' (masculine), 'unas' (feminine), 'des' (masculine/feminine). Also, colloquial English employs unstressed 'some' ["səm" or "s'm"] as an indefinite article. For example, the following are colloquially interchangeable.

> there is ∅ milk in the refrigerator
> there is **səm** milk in the refrigerator

2. Initial Hypothesis

In accounting for ‘a’, the following is a fairly natural initial hypothesis.

(iH) ‘a’ is a variant of ‘some’

$[\![\text{a}]\!] \quad = \quad [\![\text{some}]\!] \quad = \quad \lambda P_0 \vee \{x \mid Px\}$

This hypothesis accounts for the following example.

1. Jay owns a dog

Jay	[+1]	owns	a	dog	[+2]
J	$\lambda x\{x_1\}$		$\lambda P_0 \vee\{x \mid Px\}$	$\mathbf{D}_0$	
			$\vee\{ x \mid \mathbf{D}x \}$		$\lambda x\{x_2\}$
		$\lambda y_2 \lambda x_1 \mathbf{O}xy$	$\vee\{ x_2 \mid \mathbf{D}x \}$		
J_1		$\vee\{ \lambda y_1 \mathbf{O}yx \mid \mathbf{D}x \}$			
$\vee\{ \mathbf{O}Jx \mid \mathbf{D}x \}$					
$\exists x \{ \mathbf{D}x \,\&\, \mathbf{O}Jx \}$					

It also accounts for the following examples.

2. Jay owns dogs

Jay	[+1]	owns	[s'm]	dog	s [+plural]	[+2]
J	$\lambda x\{x_1\}$			$\mathbf{D}_0$	$\lambda P_0 \lambda x_0\{Px \,\&\, \mathbb{P}x\}$	
			$\lambda P_0 \vee\{x \mid Px\}$	$\lambda x_0\{\mathbf{D}x \,\&\, \mathbb{P}x\}$		
			$\vee\{ x \mid \mathbf{D}x \,\&\, \mathbb{P}x \}$			$\lambda x\{x_2\}$
		$\lambda x_2 \lambda y_1 \mathbf{O}yx$	$\vee\{ x_2 \mid \mathbf{D}x \,\&\, \mathbb{P}x \}$			
J_1		$\vee\{ \lambda y_1 \mathbf{O}yx \mid \mathbf{D}x \,\&\, \mathbb{P}x \}$				
$\vee\{ \mathbf{O}Jx \mid \mathbf{D}x \,\&\, \mathbb{P}x \}$						
$\exists x \{ \mathbf{D}x \,\&\, \mathbb{P}x \,\&\, \mathbf{O}Jx \}$						

3. Jay owns land

Jay	[+1]	owns	[s'm]	land	[+mass]	[+2]
J	$\lambda x\{x_1\}$			$\mathbf{L}_0$	$\lambda P_0 \lambda x_0\{Px \,\&\, \mathbb{M}x\}$	
			$\lambda P_0 \vee\{x \mid Px\}$	$\lambda x_0\{\mathbf{L}x \,\&\, \mathbb{M}x\}$		
			$\vee\{ x \mid \mathbf{L}x \,\&\, \mathbb{M}x \}$			$\lambda x\{x_2\}$
		$\lambda x_2 \lambda y_1 \mathbf{O}yx$	$\vee\{ x_2 \mid \mathbf{L}x \,\&\, \mathbb{M}x \}$			
J_1		$\vee\{ \lambda y_1 \mathbf{O}yx \mid \mathbf{L}x \,\&\, \mathbb{M}x \}$				
$\vee\{ \mathbf{O}Jx \mid \mathbf{L}x \,\&\, \mathbb{M}x \}$						
$\exists x \{ \mathbf{L}x \,\&\, \mathbb{M}x \,\&\, \mathbf{O}Jx \}$						

Here, we have expanded the underlying domain of entities to include plural-entities (pluralities), signified by the predicate ‘$\mathbb{P}$’, and mass-entities (quantities), signified by the predicate ‘$\mathbb{M}$’.

Then the root-word ‘dog’ is understood to apply to dog-entities, including dog-individuals, dog-pluralities, and dog-matter.[3]

We also hypothesize that, when ‘a’ attaches to a plural-noun or mass-noun, it is changed to unstressed ‘some’ [s'm], which is often deleted in the final presentation (pronunciation/spelling).

3. Problems with the Initial Hypothesis

Although (iH) accounts for our initial data, it faces serious difficulties accounting for the following examples.

> **a** dog is **a** mammal
> if **a** dog is well-fed, then it is happy
> every man who owns **a** dog feeds it
> Jay is looking for **a** dog

Applying (iH) to these examples produces the following semantic-trees, which do not yield the intended readings of these sentences.

1. a dog is a mammal

<table>
<tr><td>a dog [+1]</td><td>is</td><td>a mammal [+2]</td></tr>
<tr><td rowspan="2">$\vee\{ x_1 \mid \mathbf{D}x \}$</td><td>$\lambda y_2 \lambda x_1[x=y]$</td><td>$\vee\{ y_2 \mid \mathbf{M}y \}$</td></tr>
<tr><td colspan="2">$\vee\{ \lambda x_1[x=y] \mid \mathbf{M}y \}$</td></tr>
<tr><td colspan="3">$\vee\{ \vee\{ x=y \mid \mathbf{D}x \} \mid \mathbf{M}y \}$</td></tr>
<tr><td colspan="3">$\exists y \{ \mathbf{M}y \,\&\, \exists x\{\mathbf{D}x \,\&\, x=y\} \}$
※ $\exists x\{\mathbf{M}x \,\&\, \mathbf{D}x\}$ ※</td></tr>
</table>

2. if a dog is well-fed, then it is happy

<table>
<tr><td>if</td><td>a dog</td><td>[+1,−1]</td><td>is well-fed</td><td>then</td><td>(−1) it [+1]</td><td>is happy</td></tr>
<tr><td rowspan="3">$\lambda P \lambda Q\{P \rightarrow Q\}$</td><td>$\vee\{ x \mid \mathbf{D}x \}$</td><td>$\lambda x\{x_1 \times x_{-1}\}$</td><td rowspan="2">$\lambda x_0 \mathbf{W}x$</td><td>$\varnothing$</td><td>$\lambda x_{-1}\{x_1\}$</td><td>$\lambda x_1 \mathbf{H}x$</td></tr>
<tr><td colspan="2">$\vee\{ x_1 \times x_{-1} \mid \mathbf{D}x \}$</td><td colspan="3" rowspan="3">$\lambda x_{-1} \mathbf{H}x$</td></tr>
<tr><td colspan="3">$\vee\{ \mathbf{W}x \times x_{-1} \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="4">$\vee\{ \lambda Q\{\mathbf{W}x \rightarrow Q\} \times x_{-1} \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="7">$\vee\{ \lambda Q\{\mathbf{W}x \rightarrow Q\} \times x_{-1} \times \lambda x_{-1}\mathbf{H}x \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="7">$\vee\{ \lambda Q\{\mathbf{W}x \rightarrow Q\} \times \mathbf{H}x \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="7">$\vee\{ \mathbf{W}x \rightarrow \mathbf{H}x \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="7">※ $\exists x \{ \mathbf{D}x \,\&\, (\mathbf{W}x \rightarrow \mathbf{H}x) \}$ ※</td></tr>
</table>

[3] In some cases, the very same phonetic form conveys all three morphemes, as in ‘fish’.

3. every man who owns a dog feeds it

every	man	who [+1]	owns	a dog	[+2, −1]	[+1]	feeds	(−1) it [+2]
				$\vee\{y \mid Dy\}$	$\lambda x\{x_2 \times x_{-1}\}$		$\lambda y_2\lambda x_1 Fxy$	$\lambda z_{-1}\{z_2\}$
			$\lambda y_2\lambda x_1 Oxy$	$\vee\{ y_2 \times y_{-1} \mid Dy \}$				
		$\lambda Q_1\lambda P_0\lambda x_0$ $\{Px\&Qx\}$	$\vee\{ \lambda x_1 Oxy \times y_{-1} \mid Dy \}$					
	M_0	$\vee\{ \lambda P0\lambda x0\{Px\&Oxy\} \times y_{-1} \mid Dy \}$						
$\lambda P_0\wedge\{x \mid Px\}$	$\vee\{ \lambda x_0\{Mx\&Oxy\} \times y_{-1} \mid Dy \}$							
$\vee\{ \wedge\{x \mid Mx\&Oxy\} \times y_{-1} \mid Dy \}$						$\lambda x\{x_1\}$		
$\vee\{ \wedge\{x_1 \mid Mx\&Oxy\} \times y_{-1} \mid Dy \}$							$\lambda z_{-1\lambda} x_1 Fxz$	
$\vee\{ \wedge\{x_1 \mid Mx\&Oxy\} \times \lambda x_1 Fxy \mid Dy \}$								
$\vee\{ \wedge\{Fxy \mid Mx\&Oxy\} \mid Dy \}$								
※ $\exists y \{ Dy \,\&\, \forall x\{ \{Mx\&Oxy\} \rightarrow Fxy \} \}$ ※								

The above construction grants 'a dog' wide-scope, which allows it to bind 'it', but the resulting reading is incorrect. If we pursue a construction that grants 'every man' wide-scope, then 'a dog' fails to bind 'it'.

4. Jay is looking for a dog

Jay [+1]	is-looking-for	a dog [+2]
	$\lambda y_2\lambda x_1 \mathbf{L}xy$	$\vee\{ y_2 \mid \mathbf{D}y \}$
J_1	$\vee\{ \lambda x_1 \mathbf{L}xy \mid \mathbf{D}y \}$	
$\vee\{ \mathbf{L}Jy \mid \mathbf{D}y \}$		
$\exists y \{ \mathbf{D}y \,\&\, \mathbf{L}Jy \}$		

According to this reading, there is a (particular) dog that Jay is looking for. Although this is an admissible reading, there is another, more plausible, reading according to which Jay is not looking for a particular dog; rather, 'a dog' is better thought of as indicating the *kind* of thing Jay is looking for. This is a reading that (iH) cannot generate.

4. The New Proposal

In order to account for indefinite noun phrases, we take a three-part approach.

(1) we propose a dual-pair of junctions – *product* and *sum*.

(2) we propose a type-logical extension of our account of common-noun-phrases.

(3) we propose to treat the article 'a', not as a quantifier, but as a common-noun modifier.

1. Product and Sum

We propose a dual-pair of junctions – product Π, and sum Σ. This involves expanding the type-formation rules so that if $\mathfrak{I}$ is a type, then so are $\Pi\mathfrak{I}$ and $\Sigma\mathfrak{I}$. It also involves expanding the syntax of type-theory to include all expressions of the following forms.

$\Pi_\nu\{\varepsilon \mid \Phi\}$ the product [over ν] of all ε such that Φ
$\Sigma_\nu\{\varepsilon \mid \Phi\}$ the sum [over ν] of all ε such that Φ

Here, ν is any variable [which is often omitted], ε is any expression of type $\mathfrak{I}$, Φ is any formula, and the resulting expression has type $\Pi\mathfrak{I}$ [respectively, $\Sigma\mathfrak{I}$].

The following are the associated simplification-rules.

$\Sigma D = D$
$\Sigma S = S$
$\Sigma_\nu\{\Phi \mid \Psi\}$ / $\exists\nu\{\Psi \,\&\, \Phi\}$

$\Pi S \neq S$
$\Pi_\nu\{\Phi \mid \Psi\}$ / $\forall\nu\{\Psi \rightarrow \Phi\}$ provided node is assertoric[4]

The following are the associated composition-rules.

α	α	α
$\{\alpha,\beta\} \vdash \gamma$	$\{\alpha,\beta\} \vdash \gamma$	$\{\alpha,\beta\} \vdash \gamma$
$\Pi\{\beta \mid \Phi\}$	$\Sigma\{\beta \mid \Phi\}$	$\Sigma\{\beta \mid \Phi\}$
$\Pi\{\gamma \mid \Phi\}$	$\Pi\{\gamma \mid \Phi\}$	$\Sigma\{\gamma \mid \Phi\}$
no admissibility-restrictions	if α is *any-promoting* (e.g., if α is anti-tonic)	if α is not *any-promoting*, and α is not a junction other than Σ

2. Common-Noun-Phrases Transform into Entity-Sums

As before, we propose that common-noun-phrases are fundamentally 0-marked predicates, which is to say they have type $D_0 \rightarrow S$. We further propose that every such phrase gives rise to an associated entity-sum, in accordance with the following *transformational rule*.

$$\lambda\nu_0\Phi \quad // \quad \Sigma\{\nu \mid \Phi\}$$

This is not an identity, since the objects don't have the same type. Rather, it is a bi-directional rule that authorizes *deriving* a sum-of-entities from a nullative-predicate, and conversely.

[4] As before, assertoric is ultimately complicated. For the moment, at least, a node is assertoric if and only if it is topmost and contains no free variables. This equivalence is then based on the plausible intuition that to assert a product of sentences is to assert all those sentences.

3. The Indefinite Article ‘a’

We propose that ‘a’ is fundamentally a number-marker, semantically rendered as follows.[5]

$$[\![\mathrm{a}]\!] \quad = \quad \lambda \mathrm{P}_0 \lambda x_0 \{ \mathrm{P}x \ \& \ \mathbf{1}x \}$$

Here, ‘**1**’ is understood as follows.

$$\mathbf{1}x \quad =_{\mathrm{df}} \quad x \text{ is "one" entity}$$

which is regarded as a primitive notion.[6] If we do not admit plural-nouns or mass-nouns, as is common in elementary logic, then ‘a’ is redundant, but if we do admit these, then ‘a’ distinguishes (say)

being **a** fish (singular) from
being fish (plural), and from
being fish (mass).

5. Examples

In the following, we concentrate on singular-nouns, and accordingly treat ‘a’ as redundant.

1. every man owns a dog

<table>
<tr><td>every man [+1]</td><td>owns</td><td>a dog</td><td>[+2]</td></tr>
<tr><td rowspan="4"></td><td rowspan="3"></td><td>$\lambda y_0 \mathbf{D}y$</td><td></td></tr>
<tr><td>$\Sigma\{y \mid \mathbf{D}y\}$</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td colspan="2">$\Sigma\{ y_2 \mid \mathbf{D}y \}$</td></tr>
<tr><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td colspan="2"></td></tr>
<tr><td>$\wedge\{ x_1 \mid \mathbf{M}x \}$</td><td colspan="3">$\Sigma\{ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="4">$\wedge\{ \Sigma\{ \mathbf{O}xy \mid \mathbf{D}y \} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="4">$\forall x \{ \mathbf{M}x \rightarrow \exists y\{\mathbf{D}y \ \& \ \mathbf{O}xy\} \}$</td></tr>
</table>

<table>
<tr><td>every man [+1]</td><td>owns</td><td>a dog</td><td>[+2]</td></tr>
<tr><td rowspan="4"></td><td rowspan="2"></td><td>$\lambda y_0 \mathbf{D}y$</td><td></td></tr>
<tr><td>$\Sigma\{y \mid \mathbf{D}y\}$</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td colspan="2">$\Sigma\{ y_2 \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="3">$\Sigma\{ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td>$\wedge\{ x_1 \mid \mathbf{M}x \}$</td><td colspan="3">$\lambda x_1 \ \exists y \ \{ \mathbf{D}y \ \& \ \mathbf{O}xy \}$</td></tr>
<tr><td colspan="4">$\wedge\{ \exists y\{\mathbf{D}y \ \& \ \mathbf{O}xy\} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="4">$\forall x \{ \mathbf{M}x \rightarrow \exists y\{\mathbf{D}y \ \& \ \mathbf{O}xy\} \}$</td></tr>
</table>

[5] In many languages, including the Romance languages, the same surface form (spelling, pronunciation) is used for both the indefinite article “a” and the numeral “one”. Latin has no articles, yet all its descendents do, which is an evolutionary curiosity. It is generally believed that the indefinite articles in Romance languages derive (independently?) from the Latin word ‘unus’ for ‘one’.

[6] ‘one’ is scare-quoted because its application is heavily context-dependent. Usually, it obviously denotes simple singularity, but other times, as in

a man and woman who are married

where ‘a man and woman’ means “one man-woman-pair”, which is not a simple singularity.

2. every man who owns a dog is happy

every [+1]	man	who [+1]	owns	a dog [+2]	is happy
			$\lambda y_2 \lambda x_1 \mathbf{O}xy$	$\Sigma\{y_2 \mid \mathbf{D}y\}$	
		$\lambda Q_1 \lambda P_0 \lambda x_0\{Px \& Qx\}$	$\Sigma\{\ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y\ \}$		
	$\mathbf{M}_0$	$\Sigma\{\ \lambda P_0 \lambda x_0\{Px\ \&\ \mathbf{O}xy\} \mid \mathbf{D}y\ \}$			
$\lambda P_0 \wedge\{x_1 \mid Px\}$	$\Sigma\{\ \lambda x_0\{\mathbf{M}x\ \&\ \mathbf{O}xy\} \mid \mathbf{D}y\ \}$				
$\Pi\{\ \wedge\{x_1 \mid \mathbf{M}x\ \&\ \mathbf{O}xy\} \mid \mathbf{D}y\ \}$					$\lambda x_1 \mathbf{H}x$
$\Pi\{\ \wedge\{\mathbf{H}x \mid \mathbf{M}x\ \&\ \mathbf{O}xy\} \mid \mathbf{D}y\ \}$					
$\forall y\ \{\ \mathbf{D}y \rightarrow \forall x\{\{\mathbf{M}x\ \&\ \mathbf{O}xy\} \rightarrow \mathbf{H}x\ \}$					

In this computation, ⟦every⟧ is anti-tonic and therefore converts Σ to Π, which ultimately grants 'a dog' wide-scope. There is another computation, given as follows, according to which 'a dog' has narrow-scope, although the resulting formula is logically-equivalent to the previous example.

3. every man who owns a dog is happy [alternative computation]

every [+1]	man	who [+1]	owns a dog	is happy
			$\Sigma\{\ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y\ \}$	
		$\lambda Q_1 \lambda P_0 \lambda x_0\{Px \& Qx\}$	$\lambda x_1\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}$	
	$\mathbf{M}_0$	$\lambda P_0\ \lambda x_0\ \{\ Px\ \&\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\ \}$		
$\lambda P_0 \wedge\{x_1 \mid Px\}$	$\lambda x_0\ \{\ \mathbf{M}x\ \&\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\ \}$			
$\wedge\{\ x_1 \mid \mathbf{M}x\ \&\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\ \}$				$\lambda x_1 \mathbf{H}x$
$\wedge\{\ \mathbf{H}x \mid \mathbf{M}x\ \&\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\ \}$				
$\forall x\ \{\ \{\mathbf{M}x\ \&\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}\} \rightarrow \mathbf{H}x\ \}$				

In the previous example, 'a dog' can be narrow-$\exists$ or wide-$\forall$, the resulting formulas being logically equivalent. The following example is superficially similar, but semantically quite different.

4. every man who owns a dog feeds **it**

every [+1]	man	who	owns	a dog	[+2, −1]	feeds	(−1) it [+2]
				$\Sigma\{y \mid \mathbf{D}y\}$	$\lambda x\{x_1 \times x_{-1}\}$	$\lambda y_2 \lambda x_1 \mathbf{F}xy$	$\lambda z_{-1}\{z_2\}$
			$\lambda y_2 \lambda x_1 \mathbf{O}xy$	$\Sigma\{\ y_2 \times y_{-1} \mid \mathbf{D}y\ \}$			
		$\lambda Q_1 \lambda P_0 \lambda x_0\{Px \& Qx\}$	$\Sigma\{\ \lambda x_1 \mathbf{O}xy \times y_{-1} \mid \mathbf{D}y\ \}$				
	$\mathbf{M}_0$	$\Sigma\{\ \lambda P_0 \lambda x_0\{Px\ \&\ \mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y\ \}$					
$\lambda P_0 \wedge\{x_1 \mid Px\}$	$\Sigma\{\ \lambda x_0\{\mathbf{M}x\ \&\ \mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y\ \}$						
$\Pi\{\ \wedge\{x_1 \mid \mathbf{M}x\ \&\ \mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y\ \}$						$\lambda z_{-1} \lambda x_1 \mathbf{F}xz$	
$\Pi\{\ \wedge\{x_1 \mid \mathbf{M}x\ \&\ \mathbf{O}xy\} \times \lambda x_1 \mathbf{F}xy \mid \mathbf{D}y\ \}$							
$\Pi\{\ \wedge_x\{\mathbf{F}xy \mid \mathbf{M}x\ \&\ \mathbf{O}xy\} \mid \mathbf{D}y\ \}$							
$\forall y\ \{\ \mathbf{D}y \rightarrow \forall x\{\ \{\mathbf{M}x\ \&\ \mathbf{O}xy\} \rightarrow \mathbf{F}xy\ \}\ \}$							

Notice, in particular, that a narrow-scope reading of ‘a dog’ is impossible because Σ-simplification is not available at any node, because the sum is not a sum of sentences. In order to accomplish the latter, we must remove the anaphoric marker [–1], but then ‘a dog’ does not bind ‘it’.

The following is another example in which ‘a’ must gain wide-scope in order to bind its pronoun.

5. if a dog is well-fed, then it is happy

<table>
<tr><td>if</td><td>a dog</td><td>[+1, −1]</td><td>is well-fed</td><td>then</td><td>(−1) it [+1]</td><td>is happy</td></tr>
<tr><td rowspan="3">$\lambda P\lambda Q\{P\to Q\}$</td><td>$\Sigma\{x_1 \mid \mathbf{D}x\}$</td><td>$\lambda x\{x_1\times x_{-1}\}$</td><td rowspan="2">$\lambda x_1\mathbf{W}x$</td><td>$\varnothing$</td><td>$\lambda x_{-1}\{x_1\}$</td><td>$\lambda x_1\mathbf{H}x$</td></tr>
<tr><td colspan="2">$\Sigma\{ x_1 \times x_{-1} \mid \mathbf{D}x \}$</td><td colspan="3" rowspan="3">$\lambda x_{-1}\mathbf{H}x$</td></tr>
<tr><td colspan="3">$\Sigma\{ \mathbf{W}x \times x_{-1} \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="4">$\Pi\{ \lambda Q\{\mathbf{W}x\to Q\} \times x_{-1} \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="7">$\Pi\{ \lambda Q\{\mathbf{W}x\to Q\} \times \mathbf{H}x \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="7">$\Pi\{ \mathbf{W}x\to\mathbf{H}x \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="7">$\forall x \{ \mathbf{D}x \to \{\mathbf{W}x \to \mathbf{H}x\} \}$</td></tr>
</table>

Notice that ⟦if⟧ is anti-tonic, and accordingly converts Σ to Π. The following is a variation in which ‘if’ is centrally placed.

6. a dog is happy if it is well-fed

<table>
<tr><td>a dog</td><td>[+1, −1]</td><td>is happy</td><td>if</td><td>(−1) it [+1]</td><td>is well-fed</td></tr>
<tr><td>$\Sigma\{x \mid \mathbf{D}x\}$</td><td>$\lambda x\{x_1\times x_{-1}\}$</td><td rowspan="2">$\lambda x_1\mathbf{H}x$</td><td rowspan="2">$\lambda P\lambda Q\{P\to Q\}$</td><td>$\lambda x_{-1}\{x_1\}$</td><td>$\lambda x_1\mathbf{W}x$</td></tr>
<tr><td colspan="2">$\Sigma\{ x_1 \times x_{-1} \mid \mathbf{D}x \}$</td><td colspan="2">$\lambda x_{-1}\mathbf{W}x$</td></tr>
<tr><td colspan="3">$\Sigma\{ \mathbf{H}x \times x_{-1} \mid \mathbf{D}x \}$</td><td colspan="3">$\lambda x_{-1}\lambda Q\{\mathbf{W}x\to Q\}$</td></tr>
<tr><td colspan="6">$\Pi\{ \mathbf{H}x \times \lambda Q\{\mathbf{W}x\to Q\} \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="6">$\Pi\{ \mathbf{W}x\to\mathbf{H}x \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="6">$\forall x \{ \mathbf{D}x \to \{\mathbf{W}x \to \mathbf{H}x\} \}$</td></tr>
</table>

Notice that ⟦if it is well-fed⟧ is anti-tonic, and accordingly converts Σ to Π.

The following is a variation with two occurrences of ‘a’ with corresponding pronouns.

7. if a man owns a dog, then he feeds it

<table>
<tr><td>if</td><td>a man [+1,−1]</td><td>owns</td><td>a dog [+2, −2]</td><td>then</td><td>(−1) he [+1]</td><td>feeds</td><td>(−2) it [+2]</td></tr>
<tr><td rowspan="3">$\lambda P\lambda Q$ $\{P\to Q\}$</td><td rowspan="2">$\Sigma\{ x_1 \times x_{-1} \mid \mathbf{M}x \}$</td><td>$\lambda y_2\lambda x_1\mathbf{O}xy$</td><td>$\Sigma\{ y_2 \times y_{-2} \mid \mathbf{D}y \}$</td><td rowspan="2">$\varnothing$</td><td rowspan="2">$\lambda x_{-1}\{x_1\}$</td><td>$\lambda y_2\lambda x_1\mathbf{F}xy$</td><td>$\lambda y_{-2}\{y_2\}$</td></tr>
<tr><td colspan="2">$\Sigma\{ \lambda x_1\mathbf{O}xy \times y_{-2} \mid \mathbf{D}y \}$</td><td colspan="2">$\lambda y_{-2}\lambda x_1\mathbf{F}xy$</td></tr>
<tr><td colspan="3">$\Sigma\{ \Sigma\{ \mathbf{O}xy \times y_{-2} \mid \mathbf{D}y \} \times x_{-1} \mid \mathbf{M}x \}$</td><td colspan="4" rowspan="2">$\lambda y_{-2}\lambda x_{-1}\mathbf{F}xy$</td></tr>
<tr><td colspan="4">$\Pi\{ \Pi\{ \lambda Q\{\mathbf{O}xy\to Q\} \times y_{-2} \mid \mathbf{D}y \} \times x_{-1} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="8">$\Pi\{ \Pi\{ \mathbf{O}xy \to \mathbf{F}xy \mid \mathbf{D}y \} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="8">$\forall x \{ \mathbf{M}x \to \forall y \{ \mathbf{D}y \to \{\mathbf{O}xy\to\mathbf{F}xy\} \} \}$</td></tr>
</table>

Notice that ⟦if⟧ is anti-tonic, and accordingly converts each Σ to Π.