1. Indefinite Noun Phrases

In order to account for indefinite noun phrases, we take a three-part approach.

(1) we propose a dual-pair of junctions – *product* and *sum*.
(2) we propose a type-logical extension of our account of common-noun-phrases.
(3) we propose to treat the article 'a', not as a quantifier, but as a common-noun modifier.

1. Product and Sum

We propose a dual-pair of junctions – product Π, and sum Σ. This involves expanding the type-formation rules so that if $\mathfrak{I}$ is a type, then so are $\Pi\mathfrak{I}$ and $\Sigma\mathfrak{I}$. It also involves expanding the syntax of type-theory to include all expressions of the following forms.

$\Pi_\nu\{\varepsilon \mid \Phi\}$ the product [over ν] of all ε such that Φ
$\Sigma_\nu\{\varepsilon \mid \Phi\}$ the sum [over ν] of all ε such that Φ

Here, ν is any variable [which is often omitted], ε is any expression of type $\mathfrak{I}$, Φ is any formula, and the resulting expression has type $\Pi\mathfrak{I}$ [respectively, $\Sigma\mathfrak{I}$].

The following are the associated simplification-rules.

$\Sigma D = D$			
$\Sigma S = S$			
$\Sigma_\nu\{\Phi \mid \Psi\}$	/	$\exists\nu\{\Psi \,\&\, \Phi\}$	
$\Pi S \neq S$			
$\Pi_\nu\{\Phi \mid \Psi\}$	/	$\forall\nu\{\Psi \to \Phi\}$	provided node is topmost[1]

The following are the associated composition-rules.

α	α	α
$\{\alpha,\beta\} \vdash \gamma$	$\{\alpha,\beta\} \vdash \gamma$	$\{\alpha,\beta\} \vdash \gamma$
$\Pi\{\beta \mid \Phi\}$	$\Sigma\{\beta \mid \Phi\}$	$\Sigma\{\beta \mid \Phi\}$
$\Pi\{\gamma \mid \Phi\}$	$\Pi\{\gamma \mid \Phi\}$	$\Sigma\{\gamma \mid \Phi\}$
no admissibility-restrictions	if α is *any-promoting* (e.g., if α is anti-tonic)	if α is not *any-promoting*, and α is not a junction other than Σ

[1] This transformation is then based on the plausible intuition that to assert a product of sentences is to assert all those sentences.

2. Common-Noun-Phrases Transform into Entity-Sums

As before, we propose that a common-noun-phrase is fundamentally a 0-marked predicate, which is to say an item of type $D_0 \rightarrow S$. We further propose that every such phrase gives rise to an associated entity-sum, in accordance with the following *transformational rule*.

$$\lambda\nu_0\Phi \quad // \quad \Sigma\{\nu \mid \Phi\}$$

This is not an identity, since the objects don't have the same type. Rather, it is a bi-directional rule that authorizes *deriving* an entity-sum (type ΣD) from a nullative-predicate (type $D_0 \rightarrow S$), and conversely.

3. The Indefinite Article 'a'

We propose that 'a' is fundamentally a number-word, basically equivalent to 'one' semantically rendered as follows.[2]

$$[\![\text{a}]\!] \quad = \quad \lambda P_0 \lambda x_0\{Px \;\&\; \mathbf{1}x\}$$

Here, '**1**' is understood as follows.

$$\mathbf{1}x \quad =_{df} \quad x \text{ is unital}$$

where 'unital' is a primitive notion.[3] If we do not admit plural-nouns or mass-nouns, as is common in elementary logic, then 'a' is redundant, but if we do admit these, then 'a' distinguishes (say) among the following.

being	**a** fish	(singular-entity)
being	fish	(plural-entity)
being	fish	(mass-entity)

2. Examples

In the following, we concentrate on singular-nouns, and accordingly treat 'a' as redundant; for example, 'a dog' is equivalent to 'dog'.

1. every man who owns a dog is happy

<table>
<tr><td>every [+1]</td><td>man</td><td>who [+1]</td><td>owns</td><td>a dog [+2]</td><td>is happy</td></tr>
<tr><td></td><td></td><td></td><td>$\lambda y_2\lambda x_1\mathbf{O}xy$</td><td>$\Sigma\{y_2 \mid \mathbf{D}y\}$</td><td></td></tr>
<tr><td></td><td></td><td>$\lambda Q_1\lambda P_0\lambda x_0\{Px\&Qx\}$</td><td colspan="2">$\Sigma\{\ \lambda x_1\mathbf{O}xy \mid \mathbf{D}y\ \}$</td><td></td></tr>
<tr><td></td><td>$\mathbf{M}_0$</td><td colspan="3">$\Sigma\{\ \lambda P_0\lambda x_0\{Px \;\&\; \mathbf{O}xy\} \mid \mathbf{D}y\ \}$</td><td></td></tr>
<tr><td>$\lambda P_0\bigwedge\{x_1 \mid Px\}$</td><td colspan="4">$\Sigma\{\ \lambda x_0\{\mathbf{M}x \;\&\; \mathbf{O}xy\} \mid \mathbf{D}y\ \}$</td><td></td></tr>
<tr><td colspan="5">$\Pi\{\ \bigwedge\{x_1 \mid \mathbf{M}x \;\&\; \mathbf{O}xy\} \mid \mathbf{D}y\ \}$</td><td>$\lambda x_1\mathbf{H}x$</td></tr>
<tr><td colspan="6">$\Pi\{\ \bigwedge\{\mathbf{H}x \mid \mathbf{M}x \;\&\; \mathbf{O}xy\} \mid \mathbf{D}y\ \}$</td></tr>
<tr><td colspan="6">$\forall y\ \{\ \mathbf{D}y \rightarrow \forall x\{\{\mathbf{M}x \;\&\; \mathbf{O}xy\} \rightarrow \mathbf{H}x\ \}$</td></tr>
</table>

[2] Many languages use the very same word for both 'a' and 'one'. See chapter "Numerical Quantifiers". Note that 'one' can be used as a bare-adjective, whereas 'a' cannot.

[3] Also, its application is heavily context-dependent. Usually, "units" are simple individuals (another primitive notion), but other times they are more complex entities, as in the following,

a man and woman who are married

where 'a man and woman' means "one man-woman pair", which is not a simple individual.

2. every man who owns a dog is happy [alternative computation]

<table>
<tr><td>every [+1]</td><td>man</td><td>who [+1]</td><td>owns a dog</td><td>is happy</td></tr>
<tr><td rowspan="4">$\lambda P_0 \wedge\{x_1 \mid Px\}$</td><td rowspan="3">$\mathbf{M}_0$</td><td rowspan="2">$\lambda Q_1 \lambda P_0 \lambda x_0\{Px \& Qx\}$</td><td>$\Sigma\{ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y \}$</td><td rowspan="5">$\lambda x_1 \mathbf{H}x$</td></tr>
<tr><td>$\lambda x_1\ \exists y\{\mathbf{D}y \ \&\ \mathbf{O}xy\}$</td></tr>
<tr><td colspan="2">$\lambda P_0\ \lambda x_0\ \{\ Px \ \&\ \exists y\{\mathbf{D}y \ \&\ \mathbf{O}xy\}\ \}$</td></tr>
<tr><td colspan="3">$\lambda x_0\ \{\ \mathbf{M}x \ \&\ \exists y\{\mathbf{D}y \ \&\ \mathbf{O}xy\}\ \}$</td></tr>
<tr><td colspan="4">$\wedge\{\ x_1 \mid \mathbf{M}x \ \&\ \exists y\{\mathbf{D}y \ \&\ \mathbf{O}xy\}\ \}$</td></tr>
<tr><td colspan="5">$\wedge\{\ \mathbf{H}x \mid \mathbf{M}x \ \&\ \exists y\{\mathbf{D}y \ \&\ \mathbf{O}xy\}\ \}$</td></tr>
<tr><td colspan="5">$\forall x\ \{\ \{\mathbf{M}x \ \&\ \exists y\{\mathbf{D}y \ \&\ \mathbf{O}xy\}\} \rightarrow \mathbf{H}x\ \}$</td></tr>
</table>

In the previous example, 'a dog' can be narrow-∃ or wide-∀, the resulting formulas being logically equivalent. The following example is superficially similar, but semantically quite different.

3. every man who owns a dog feeds **it**

<table>
<tr><td>every [+1]</td><td>man</td><td>who [+1]</td><td>owns</td><td>a dog [+2,–1]</td><td>feeds</td><td>(–1) it [+2]</td></tr>
<tr><td rowspan="4">$\lambda P_0\ \wedge\{x_1 \mid \mathbf{M}x\}$</td><td rowspan="3">$\mathbf{M}_0$</td><td rowspan="2">$\lambda Q_1 \lambda P_0 \lambda x_0\{Px \& Qx\}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{y_2 \times y_{-1} \mid \mathbf{D}y\}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{F}xy$</td><td>$\lambda y_{-1}\{y_2\}$</td></tr>
<tr><td colspan="2">$\Sigma\{\ \lambda x_1 \mathbf{O}xy \times y_{-1} \mid \mathbf{D}y\ \}$</td><td colspan="2" rowspan="4">$\lambda y_{-1} \lambda x_1 \mathbf{F}xy$</td></tr>
<tr><td colspan="3">$\Sigma\{\ \lambda P_0 \lambda x_0\{Px \& \mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y\ \}$</td></tr>
<tr><td colspan="4">$\Sigma\{\ \lambda x_0\{\mathbf{M}x \& \mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y\ \}$</td></tr>
<tr><td colspan="5">$\Pi\{\ \wedge\{x_1 \mid \mathbf{M}x \& \mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y\ \}$</td></tr>
<tr><td colspan="7">$\Pi\{\ \wedge\{x_1 \mid \mathbf{M}x \& \mathbf{O}xy\} \times \lambda x_1 \mathbf{F}xy \mid \mathbf{D}y\ \}$</td></tr>
<tr><td colspan="7">$\Pi\{\ \wedge\{\mathbf{F}xy \mid \mathbf{M}x \& \mathbf{O}xy\} \mid \mathbf{D}y\ \}$</td></tr>
<tr><td colspan="7">$\forall y\ \{\ \mathbf{D}y \rightarrow \forall x\{\mathbf{M}x \& \mathbf{O}xy\ . \!\rightarrow \mathbf{F}xy\}\ \}$</td></tr>
</table>

Notice that ⟦every⟧ is anti-tonic, and accordingly converts Σ to Π. The following is another example in which 'a' must gain wide-scope in order to bind its pronoun.

4. if a dog is well-fed, then it is happy

<table>
<tr><td>if</td><td>a dog [+1,–1]</td><td>is well–fed</td><td>then</td><td>(–1) it [+1]</td><td>is happy</td></tr>
<tr><td rowspan="2">$\lambda P \lambda Q\{P \rightarrow Q\}$</td><td>$\Sigma\{\ x_1 \times x_{-1} \mid \mathbf{D}x\ \}$</td><td>$\lambda x_1 \mathbf{W}x$</td><td>$\varnothing$</td><td>$\lambda x_{-1}\{x_1\}$</td><td>$\lambda x_1 \mathbf{H}x$</td></tr>
<tr><td colspan="2">$\Sigma\{\ \mathbf{W}x \times x_{-1} \mid \mathbf{D}x\ \}$</td><td colspan="3" rowspan="2">$\lambda x_{-1}\ \mathbf{H}x$</td></tr>
<tr><td colspan="3">$\Pi\{\ \lambda Q\{\mathbf{W}x \rightarrow Q\} \times x_{-1} \mid \mathbf{D}x\ \}$</td></tr>
<tr><td colspan="6">$\Pi\{\ \lambda Q\{\mathbf{W}x \rightarrow Q\} \times \mathbf{H}x \mid \mathbf{D}x\ \}$</td></tr>
<tr><td colspan="6">$\Pi\{\ \mathbf{W}x \rightarrow \mathbf{H}x \mid \mathbf{D}x\ \}$</td></tr>
<tr><td colspan="6">$\forall x\ \{\ \mathbf{D}x \rightarrow \{\mathbf{W}x \rightarrow \mathbf{H}x\}\ \}$</td></tr>
</table>

Notice that ⟦if⟧ is anti-tonic, and accordingly converts Σ to Π.

The following is a variation in which ‘if’ is centrally placed.

5. a dog is happy if it is well-fed

<table>
<tr><td>a dog [+1,–1]</td><td>is happy</td><td>if</td><td>(–1) it [+1]</td><td>is well-fed</td></tr>
<tr><td>$\Sigma\{ x_1 \times x_{-1} \mid \mathbf{D}x \}$</td><td>$\lambda x_1 \mathbf{H}x$</td><td rowspan="2">$\lambda P \lambda Q\{P \rightarrow Q\}$</td><td>$\lambda x_{-1}\{x_1\}$</td><td>$\lambda x_1 \mathbf{W}x$</td></tr>
<tr><td colspan="2" rowspan="2">$\Sigma\{ \mathbf{H}x \times x_{-1} \mid \mathbf{D}x \}$</td><td colspan="2">$\lambda x_{-1}\ \mathbf{W}x$</td></tr>
<tr><td colspan="3">$\lambda x_{-1}\ \lambda Q\ \{\mathbf{W}x \rightarrow Q\}$</td></tr>
<tr><td colspan="5">$\Pi\{ \mathbf{H}x \times \lambda Q\ \{\mathbf{W}x \rightarrow Q\} \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="5">$\Pi\{ \mathbf{W}x \rightarrow \mathbf{H}x \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="5">$\forall x\ \{ \mathbf{D}x \rightarrow \{\mathbf{W}x \rightarrow \mathbf{H}x\} \}$</td></tr>
</table>

Notice that ⟦if it is well-fed⟧ is anti-tonic, and accordingly converts Σ to Π. The following is a variation with ‘if and only if’ instead of ‘if’.

6. a dog is happy **if and only if** it is well-fed

<table>
<tr><td>a dog [+1,–1]</td><td>is happy</td><td>if-and-only-if</td><td>(–1) it [+1]</td><td>is well-fed</td></tr>
<tr><td>$\Sigma\{ x_1 \times x_{-1} \mid \mathbf{D}x \}$</td><td>$\lambda x_1 \mathbf{H}x$</td><td rowspan="2">$\lambda P \lambda Q\{P \leftrightarrow Q\}$</td><td>$\lambda x_{-1}\{x_1\}$</td><td>$\lambda x_1 \mathbf{W}x$</td></tr>
<tr><td colspan="2" rowspan="2">$\Sigma\{ \mathbf{H}x \times x_{-1} \mid \mathbf{D}x \}$</td><td colspan="2">$\lambda x_{-1}\ \mathbf{W}x$</td></tr>
<tr><td colspan="3">$\lambda x_{-1}\ \lambda Q\ \{\mathbf{W}x \leftrightarrow Q\}$</td></tr>
<tr><td colspan="5">$\Pi\{ \mathbf{H}x \times \lambda Q\ \{\mathbf{W}x \leftrightarrow Q\} \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="5">$\Pi\{ \mathbf{W}x \leftrightarrow \mathbf{H}x \mid \mathbf{D}x \}$</td></tr>
<tr><td colspan="5">$\forall x\ \{ \mathbf{D}x \rightarrow \{\mathbf{W}x \leftrightarrow \mathbf{H}x\} \}$</td></tr>
</table>

Notice that ⟦if and only if it is well-fed⟧ is not (obviously) anti-tonic; we nevertheless postulate that it is enough like ⟦if it is well-fed⟧ that it also converts Σ to Π. The following is a variation with two occurrences of ‘a’ with corresponding pronouns.

7. if a man owns a dog, then he feeds it

<table>
<tr><td>if</td><td>a man [+1,–1]</td><td>owns</td><td>a dog [+2, –2]</td><td>then</td><td>(–1) he [+1]</td><td>feeds</td><td>(–2) it [+2]</td></tr>
<tr><td rowspan="3">$\lambda P \lambda Q$ $\{P \rightarrow Q\}$</td><td rowspan="2">$\Sigma\{ x_1 \times x_{-1} \mid \mathbf{M}x \}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{ y_2 \times y_{-2} \mid \mathbf{D}y \}$</td><td rowspan="2">$\varnothing$</td><td rowspan="2">$\lambda x_{-1}\{x_1\}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{F}xy$</td><td>$\lambda y_{-2}\{y_2\}$</td></tr>
<tr><td colspan="2">$\Sigma\{ \lambda x_1 \mathbf{O}xy \times y_{-2} \mid \mathbf{D}y \}$</td><td colspan="2">$\lambda y_{-2} \lambda x_1 \mathbf{F}xy$</td></tr>
<tr><td colspan="3">$\Sigma\{ \Sigma\{ \mathbf{O}xy \times y_{-2} \mid \mathbf{D}y \} \times x_{-1} \mid \mathbf{M}x \}$</td><td colspan="4" rowspan="2">$\lambda y_{-2} \lambda x_{-1} \mathbf{F}xy$</td></tr>
<tr><td colspan="4">$\Pi\{ \Pi\{ \lambda Q\{\mathbf{O}xy \rightarrow Q\} \times y_{-2} \mid \mathbf{D}y \} \times x_{-1} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="8">$\Pi\{ \Pi\{ \lambda Q\{\mathbf{O}xy \rightarrow Q\} \times \lambda x_{-1} \mathbf{F}xy \mid \mathbf{D}y \} \times x_{-1} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="8">$\Pi\{ \Pi\{ \lambda Q\{\mathbf{O}xy \rightarrow Q\} \times \mathbf{F}xy \mid \mathbf{D}y \} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="8">$\Pi\{ \Pi\{ \mathbf{O}xy \rightarrow \mathbf{F}xy \mid \mathbf{D}y \} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="8">$\forall x\ \{ \mathbf{M}x \rightarrow \forall y\ \{ \mathbf{D}y \rightarrow \{\mathbf{O}xy \rightarrow \mathbf{F}xy\} \} \}$</td></tr>
</table>

Notice that ⟦if⟧ is anti-tonic, and accordingly converts each Σ to Π.

The following is a variation employing ‘no’.

8. if no person owns a dog, then it is free

<table>
<tr><td>if</td><td>no person [+1]</td><td>owns</td><td>a dog [+2, –1]</td><td>then</td><td>(–1) it [+1]</td><td>is free</td></tr>
<tr><td rowspan="4">$\lambda P \lambda Q\{P \rightarrow Q\}$</td><td rowspan="2">$\wedge\{\neg \times x_1 \mid \mathbf{P}x\}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{y_2 \times y_{-1} \mid \mathbf{D}y\}$</td><td>$\varnothing$</td><td>$\lambda y_{-1}\{y_1\}$</td><td>$\lambda x_1 \mathbf{F}x$</td></tr>
<tr><td colspan="2">$\Sigma\{ \lambda x_1 \mathbf{O}xy \times y_{-1} \mid \mathbf{D}y \}$</td><td colspan="3" rowspan="4">$\lambda y_{-1} \mathbf{F}y$</td></tr>
<tr><td colspan="3">$\Pi\{ \wedge\{\sim \mathbf{O}xy \mid \mathbf{P}x\} \times y_{-1} \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="3">$\Pi\{ \forall x\{\mathbf{P}x \rightarrow \sim \mathbf{O}xy\} \times y_{-1} \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="4">$\Pi\{ \forall Q\{\forall x\{\mathbf{P}x \rightarrow \sim \mathbf{O}xy\} \rightarrow Q\} \times y_{-1} \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="7">$\Pi\{ \forall Q\{\forall x\{\mathbf{P}x \rightarrow \sim \mathbf{O}xy\} \rightarrow Q\} \times \mathbf{F}y \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="7">$\Pi\{ \forall x\{\mathbf{P}x \rightarrow \sim \mathbf{O}xy\} \rightarrow \mathbf{F}y \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="7">$\forall y\{ \mathbf{D}y \rightarrow \{\forall x\{\mathbf{P}x \rightarrow \sim \mathbf{O}xy\} \rightarrow \mathbf{F}y\} \}$</td></tr>
</table>

Notice that ⟦no person⟧ is anti-tonic, and promotes Σ to Π, which admits ⟦if⟧. This is why Σ does not promote Σ to ∧ – for then it could not out-scope ‘if’. See later section on difference between ‘a’ and ‘any’.[4]

3. Sometimes ‘some’ is Indefinite

When applied to special domains, quantifier phrases occasionally take on special forms, as in ‘always’, ‘never’, ‘everywhere’, and ‘somewhere’. When the special domain is persons, we have the following forms.[5]

everyone	$=_{df}$	every person
anyone	$=_{df}$	any person
someone	$=_{df}$	some person
no one	$=_{df}$	no person

The morphological rule is clear. However, when we apply it to ‘a person’, we obtain ‘a one’, which is inadmissible.[6] What we have instead is the following abbreviation.

someone $=_{df}$ a person

Thus, ‘someone’ is ambiguous between an indefinite noun phrase and a quantifier phrase, which can be a source of semantic confusion. The following illustrates its use as an indefinite noun phrase.

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[4] Also note that, although ∧ admits Σ, it does not admit ⟦if⟧, so what might be a natural alternative derivation does not complete successfully.

[5] Note that the phrase ‘every one’ – with a slight pause between ‘every’ and ‘one’ – is analogous to ‘this one’, as in the following example.

I have many dogs; every **one** is smart; this **one** is very smart

[6] Also, the rule does not apply to plural quantifiers; for example, ‘several persons’ does not abbreviate as ‘several ones’; also, the rule does not apply to numerical quantifiers; for example, ‘exactly one person’ does not abbreviate as ‘exactly one one’.

4. The Difference between ‘a’ and ‘any’

There is a striking similarity between ‘a’ and ‘any’. In particular, both can be promoted to a junction that ultimately gets simplified to $\forall$. Because of this, the following sentences convey the same information.

> if **a** wild animal[7] comes into the house, we put it back outside
> if **any** wild animal comes into the house, we put it back outside

On the other hand, ‘a’ and ‘any’ are not equivalent, as immediately seen in the following contrast-pair.

> ☺ Rex is a dog
> ☹ Rex is any dog

There are also more complicated examples.

> if Jay doesn't respect **a woman**, he doesn't talk to **her**
> if Jay doesn't respect **any woman**, he doesn't talk to **her**

Note, in particular, that the former, but not the latter, succeeds in binding ‘her’ to the NP.

The following indicates a further difference between ‘a’ and ‘any’.

> if **a man** doesn't respect **a woman**, **he** doesn't talk to **her**
> ☹ if **any man** doesn't respect **any woman**, he doesn't talk to her

The former is sensible (grammatically); the latter is downright bizarre.

5. Scope-Issues involving Indefinite Noun Phrases

So far, we have adjudicated quantifier-scope by appealing to admissibility-restrictions on the relevant junctions. For example, $\wedge$ and $\vee$ admit each other, so ‘every’ and ‘some’ can each out-scope the other. On the other hand, $\wedge\neg$ admits $\wedge$, so ‘no’ out-scopes ‘every’, but not vice versa.

How does this idea develop when dealing with indefinite noun phrases and Σ? For example, the following sentence

> Jay doesn't own a dog

does not have a reading according to which ‘a dog’ is a wide-scope existential. The following are two admissible calculations, which produce different (but logically-equivalent) formulas.

<table>
<tr><td>Jay [+1]</td><td>doesn't</td><td>own</td><td>a dog [+2]</td></tr>
<tr><td rowspan="3">J_1</td><td rowspan="2">$\lambda P_1\lambda x_1{\sim}Px$</td><td>$\lambda y_2\lambda x_1\mathbf{O}xy$</td><td>$\Sigma\{y_2 \mid \mathbf{D}y\}$</td></tr>
<tr><td colspan="2">$\Sigma\{ \lambda x_1\mathbf{O}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="3">$\Pi\{ \lambda x_1{\sim}\mathbf{O}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="4">$\Pi\{ {\sim}\mathbf{O}\mathrm{J}y \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="4">$\forall y \{ \mathbf{D}y \rightarrow {\sim}\mathbf{O}\mathrm{J}y \}$</td></tr>
</table>

<table>
<tr><td>Jay [+1]</td><td>doesn't</td><td>own</td><td>a dog [+2]</td></tr>
<tr><td rowspan="3">J_1</td><td rowspan="2">$\lambda P_1\lambda x_1{\sim}Px$</td><td>$\lambda y_2\lambda x_1\mathbf{O}xy$</td><td>$\Sigma\{y_2 \mid \mathbf{D}y\}$</td></tr>
<tr><td colspan="2">$\Sigma\{ \lambda x_1\mathbf{O}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="2">$\lambda x_1\ \exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}$</td></tr>
<tr><td></td><td colspan="3">$\lambda x_1\ {\sim}\exists y\{\mathbf{D}y\ \&\ \mathbf{O}xy\}$</td></tr>
<tr><td colspan="4">${\sim}\exists y\{\mathbf{D}y\ \&\ \mathbf{O}\mathrm{J}y\}$</td></tr>
</table>

[7] For example, a moth!

The following is similar.

9. every man owns a dog

<table>
<tr><td>every man [+1]</td><td>owns</td><td>a dog [+2]</td></tr>
<tr><td rowspan="2">$\wedge\{x_1 \mid \mathbf{M}x\}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{y_2 \mid \mathbf{D}y\}$</td></tr>
<tr><td colspan="2">$\Sigma\{ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="3">$\wedge\{ \Sigma\{ \mathbf{O}xy \mid \mathbf{D}y \} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="3">$\forall x \{ \mathbf{M}x \rightarrow \exists y\{\mathbf{D}y \,\&\, \mathbf{O}xy\} \}$</td></tr>
</table>

<table>
<tr><td>every man [+1]</td><td>owns</td><td>a dog [+2]</td></tr>
<tr><td rowspan="3">$\wedge\{x_1 \mid \mathbf{M}x\}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{y_2 \mid \mathbf{D}y\}$</td></tr>
<tr><td colspan="2">$\Sigma\{ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="2">$\lambda x_1\ \exists y\{\mathbf{D}y \,\&\, \mathbf{O}xy\}$</td></tr>
<tr><td colspan="3">$\wedge\{ \exists y\{\mathbf{D}y \,\&\, \mathbf{O}xy\} \mid \mathbf{M}x \}$</td></tr>
<tr><td colspan="3">$\forall x \{ \mathbf{M}x \rightarrow \exists y\{\mathbf{D}y \,\&\, \mathbf{O}xy\} \}$</td></tr>
</table>

In contrast to this reading, the following reading is illegitimate.

10. every man owns a dog [illegitimate reading]

<table>
<tr><td>every man [+1]</td><td>owns</td><td>a dog [+2]</td></tr>
<tr><td rowspan="2">$\wedge\{x_1 \mid \mathbf{M}x\}$</td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td>$\Sigma\{y_2 \mid \mathbf{D}y\}$</td></tr>
<tr><td colspan="2">$\Sigma\{ \lambda x_1 \mathbf{O}xy \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="3">$\Sigma\{ \wedge\{ \mathbf{O}xy \mid \mathbf{M}x \} \mid \mathbf{D}y \}$</td></tr>
<tr><td colspan="3">※ $\exists y \{ \mathbf{D}y \,\&\, \forall x\{\mathbf{M}x \rightarrow \mathbf{O}xy\} \}$ ※</td></tr>
</table>

To block this reading, we postulate that Σ does not admit any junction except Σ, so it does not admit ∧.[8]

But then, how do we explain the following Rodney Dangerfield joke?

I have **a** suit for **every** occasion… unfortunately, this is it.

Or the following variation?

I have **a** friend in **every** city… he is very big.

If Σ does not admit ∧, how does 'a' manage to out-scope 'every', as it does in these examples?

Central to solving this problem is the idea that an indefinite noun phrase starts life as a common-noun phrase, which sometimes metamorphoses into an entity-sum. Moreover, between inception and metamorphosis, it can undergo other transformations – in particular, it can be adjectively-modified, as illustrated in the following example.

[8] This is not to say that Σ does not admit expressions that involve junctions; so Σ admits ⟦every⟧, but not ⟦every man⟧.

11. Rodney has a suit for every occasion

Rodney [+1]	has	a suit	[mod]	for	every occasion [+2]	[+2]
				$\lambda z_2 \lambda y_0 \mathbf{F}yz$	$\bigwedge\{z_2 \mid \mathbf{O}z\}$	
				$\bigwedge\{\ \lambda y_0 \mathbf{F}yz \mid \mathbf{O}z\ \}$		
			$\lambda Q_0 \lambda P_0 \lambda x_0\{Px \& Qx\}$	$\lambda y_0\ \forall z\{\mathbf{O}z \rightarrow \mathbf{F}yz\}$		
		$\mathbf{S}_0$	$\lambda P_0\ \lambda y_0\ \{\ Py\ \&\ \forall z\{\mathbf{O}z \rightarrow \mathbf{F}yz\}\ \}$			
		$\lambda y_0\ \{\ \mathbf{S}y\ \&\ \forall z\{\mathbf{O}z \rightarrow \mathbf{F}yz\}\ \}$				
		$\Sigma\{\ y \mid \mathbf{S}y\ \&\ \forall z\{\mathbf{O}z \rightarrow \mathbf{F}yz\}\ \}$				$\lambda x\{x_2\}$
	$\lambda y_2 \lambda x_1 \mathbf{H}xy$	$\Sigma\{\ y_2 \mid \mathbf{S}y\ \&\ \forall z\{\mathbf{O}z \rightarrow \mathbf{F}yz\}\ \}$				
	$\lambda x_1\ \Sigma\{\ \mathbf{H}xy \mid \mathbf{S}y\ \&\ \forall z\{\mathbf{O}z \rightarrow \mathbf{F}yz\}\ \}$					
R$_1$	$\lambda x_1\ \exists y\ \{\ \mathbf{S}y\ \&\ \forall z(\mathbf{O}z \rightarrow \mathbf{F}yz)\ \&\ \mathbf{H}xy\ \}$					
$\exists y\ \{\ \mathbf{S}y\ \&\ \forall z(\mathbf{O}z \rightarrow \mathbf{F}yz)\ \&\ \mathbf{H}\mathrm{R}y\ \}$						

As usual, we treat 'a' as redundant, being semantically absorbed by 'suit'.

Finally, compare the above tree with the following, according to which 'every' gains wide scope.

12. Rodney has a suit for every occasion [with 'every occasion' having wide scope]

Rodney [+1]	has	a suit	[mod]	for	every occasion [+2]	[+2]
				$\lambda z_2 \lambda y_0 \mathbf{F}yz$	$\bigwedge\{z_2 \mid \mathbf{O}z\}$	
			$\lambda Q_0 \lambda P_0 \lambda x_0\{Px \& Qx\}$	$\bigwedge\{\ \lambda y_0 \mathbf{F}yz \mid \mathbf{O}z\ \}$		
		$\mathbf{S}_0$	$\bigwedge\{\ \lambda P_0 \lambda y_0\{Py\ \&\ \mathbf{F}yz\} \mid \mathbf{O}z\ \}$			
		$\bigwedge\{\ \lambda y_0\{\mathbf{S}y\ \&\ \mathbf{F}yz\} \mid \mathbf{O}z\ \}$				$\lambda x\{x_2\}$
	$\lambda y_2 \lambda x_1 \mathbf{H}xy$	$\bigwedge\{\ \Sigma\{y_2 \mid \mathbf{S}y\ \&\ \mathbf{F}yz\} \mid \mathbf{O}z\ \}$				
R$_1$	$\bigwedge\{\ \Sigma\{\lambda x_1 \mathbf{H}xy \mid \mathbf{S}y\ \&\ \mathbf{F}yz\} \mid \mathbf{O}z\ \}$					
$\bigwedge\{\ \Sigma\{\mathbf{H}\mathrm{R}y \mid \mathbf{S}y\ \&\ \mathbf{F}yz\} \mid \mathbf{O}z\ \}$						
$\forall z\ \{\ \mathbf{O}z \rightarrow \exists y\ \{\ \mathbf{S}y\ \&\ \mathbf{F}yz\ \&\ \mathbf{H}\mathrm{R}y\ \}\ \}$						