1. Number-Words

By a *number-word*, or *numeral*,[1] we mean a word (or word-like compound[2]) that denotes a number. In English, numerals *appear* to be used as quantifiers, as in the following examples.

> **two** dogs are barking
> Jay owns **three** dogs
> there are **four** dogs in the yard

On the other hand, numerals also figure in the following sorts of constructions.

> the **three** dogs
> my **four** dogs
> all **five** dogs
> no **two** dogs are exactly alike
> we **three** kings (of Orient are bearing gifts we traverse afar…)

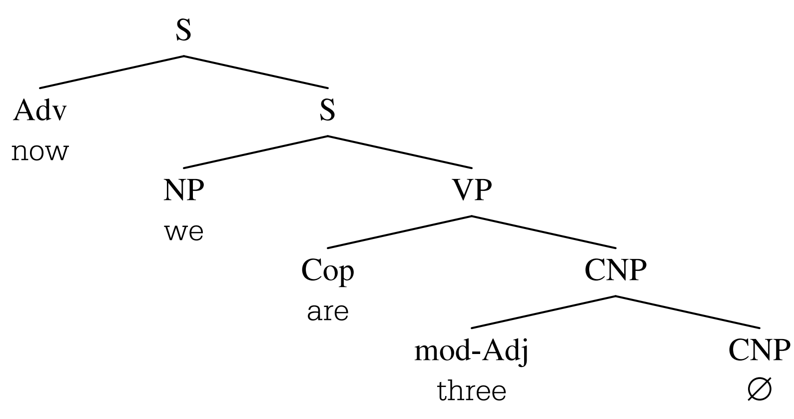
In order to account for these data, we propose that number-words are fundamentally adjectives, and the first three examples involve indefinite noun phrases, being on a par with the following parallel sentences.

> **a** dog is barking
> Jay owns **a** dog
> there is **a** dog in the yard

In particular, indefinite noun phrases can be promoted to entity-sums (type ΣD) and accordingly be used as NPs.

2. Numerical-Adjectives

One might be inclined to treat numerals as **bare-adjectives** [type C]. But given that numerals are used in combination with measure-nouns (like 'acres' and 'gallons'), and given that counting really only makes sense relative to a sortal-noun, which provides the criterion of identity and individuation, it is better to treat numerals as **modifier-adjectives** [type C→C] that are moreover *not conjunctive*.[3] In that case, the poetic 'now we are three' must be parsed as follows.



---

[1] The term 'numeral' is often reserved for **writing systems** – in particular, to refer to special logograms that denote numbers, including Arabic numerals like '11', and Roman numerals like 'II'. Also, we have decimal-numerals, hexadecimal-numerals, and binary-numerals (in wide use). So the minimalist logogram 'II' denotes two, eleven, seventeen, or three, depending upon the underlying numeration-system.

[2] We are principally interested in *basic* (but not atomic) number-words, which is to say the words and word-like compounds that children proudly recite – including, for example:

> one, fifty, two hundred, three thousand sixty-five,
> six million three hundred thousand one hundred twenty five

We take for granted the morphology of basic number-words, which is a topic unto itself, which is dealt with in the chapter "The Morphology of Number Words".

[3] So 'α is three gallons of water' does not mean 'α is three and α is gallons and α is (i.e., consists) of water', although it does mean 'α is three-gallons and α is (i.e., consists) of water'.

In particular, the presence of the null-CNP means that the sortal concept relative to which the counting is done is understood – in this case, to be persons.

With this in mind, we offer the following lexical entries.

$⟦\text{one}⟧ = \lambda P_0 \lambda x_0\, \mathbf{1}(P)[x]$
$⟦\text{two}⟧ = \lambda P_0 \lambda x_0\, \mathbf{2}(P)[x]$
etc.

Here the numerical expressions in the meta-language (alternatively, target-language) are understood as follows.

$\mathbf{1}(P)[\alpha]$	=:	α is one P
$\mathbf{2}(P)[\alpha]$	=:	α is/are[4] two P
$\mathbf{3}(P)[\alpha]$	=:	α is/are three P
etc.		

What counts as one P – for example, one person, one electron, one acre, or one mile – is taken to be primitive. On the other hand, what counts as two P, three P, etc., is logically constrained by the following.

Fundamental Measurement Principle

$(m+n)(P)[\alpha]$ iff $\exists x \exists y \{ x \perp y \;\&\; \alpha = x \oplus y \;\&\; m(P)[x] \;\&\; n(P)[y] \}$

Here, α, x, and y are entities, $\perp$ is mereological-disjointness, and $\oplus$ is mereological-sum. Also, m and n are natural numbers, if P measures count-objects, and m and n are rational numbers,[5] if P measures mass-objects; [for example, 2½ acres makes sense, but 2½ persons does not.[6]]

So, for example,

$(2+3)(\text{persons})[\alpha]$ iff
$\exists x \exists y \{ x \perp y \;\&\; \alpha = x \oplus y \;\&\; 2(\text{persons})[x] \;\&\; 3(\text{persons})[y] \}$

$(2 + ½)(\text{acres})[\alpha]$ iff
$\exists x \exists y \{ x \perp y \;\&\; \alpha = x \oplus y \;\&\; 2(\text{acres})[x] \;\&\; ½(\text{acres})[y] \}$

So, α is a 5-person plurality iff α consists of a 2-person sub-plurality and a 3-person sub-plurality disjoint from each other. Similarly, α is a 2-and-one-half-acre area iff α consists of a 2-acre sub-area and a half-acre sub-area disjoint from each other.

---

[4] The use of 'is' versus 'are' is dictated by whether P is a count-measure-noun (e.g., 'persons') or a mass-measure-noun (e.g., 'acres'). The underlying difference between persons (or planets, or electrons) and acres (or calories, or gallons) is that the former, but not the latter, are "true" individuals. Counting persons is quite different from "counting" calories.

[5] Rational numbers suffice for actual measurements, but real numbers (rational numbers plus irrational numbers) are much easier to work with theoretically.

[6] There is a TV show called "Two and a Half Men", which of course is a deliberately non-standard (and hence humorous) use of measurement words. The featured family consists of two men plus one "half-man" (which is to say, one boy).

3. Examples

1. those are three dogs

those	[+1]	are	three	dog	s	
δ	$\lambda x\{x_1\}$			$\mathbf{D}_0$	$\lambda P_0\lambda x_0\{Px\&\mathbb{P}x\}$	
			$\lambda P_0\lambda x_0\mathbf{3}(P)[x]$	$\lambda x_0\{\mathbf{D}x\&\mathbb{P}x\}$		①
		$\lambda P_0\{P_1\}$	$\lambda x_0\mathbf{3}(\mathbf{D})[x]$			②
δ_1		$\lambda x_1\mathbf{3}(\mathbf{D})[x]$				
$\mathbf{3}(\mathbf{D})[\delta]$						

Note that ① involves applying the plural-modifier to ⟦dog⟧, the latter of which consists of all dog-entities, including dog-individuals, dog-pluralities, and dog-masses (in principle). Also, note that the predicate $\mathbb{P}$ becomes redundant when combined with the **3**-predicate to produce ②.[7] The output node says that δ is a dog-trio, which is to say a plurality consisting of three dogs, which we propose henceforth to abbreviate as follows.

$$3D\delta \;=_{df}\; \mathbf{3}(\mathbf{D})[\delta] \qquad [\text{ more generally: } \mathbf{N}P\alpha \;=_{df}\; \mathbf{N}(P)[\alpha]\ ]$$

In the following examples, 'three dogs' behaves like a QP.

2. three dogs are barking

three	dogs	[+1]	are barking
$\lambda P_0\lambda x_0\mathbf{3}(P)[x]$	$\lambda x_0\{\mathbf{D}x\&\mathbb{P}x\}$		
$\lambda x_0\{\mathbf{3D}x\}$			
$\Sigma\{\ x \mid \mathbf{3D}x\ \}$		$\lambda x\{x_1\}$	
$\Sigma\{\ x_1 \mid \mathbf{3D}x\ \}$			$\lambda x_1\mathbf{B}x$
$\Sigma\{\ \mathbf{B}x \mid \mathbf{3D}x\ \}$			
$\exists x\ \{\ \mathbf{3D}x\ \&\ \mathbf{B}x\ \}$			

3. Jay owns three dogs

Jay [+1]	owns	three	dogs	[+2]
		$\lambda P_0\lambda x_0\mathbf{3}(P)[x]$	$\lambda x_0\{\mathbf{D}x\ \&\ \mathbb{P}x\}$	
		$\lambda x_0\{\mathbf{3D}x\}$		
		$\Sigma\{\ y \mid \mathbf{3D}y\ \}$		$\lambda x\{x_2\}$
	$\lambda y_2\lambda x_1\mathbf{O}xy$	$\Sigma\{\ y_2 \mid \mathbf{3D}y\ \}$		
J_1	$\Sigma\{\ \lambda x_1\mathbf{O}xy \mid \mathbf{3D}y\ \}$			
$\Sigma\{\ \mathbf{O}\mathrm{J}y \mid \mathbf{3D}y\ \}$				
$\exists y\ \{\ \mathbf{3D}y\ \&\ \mathbf{O}\mathrm{J}y\ \}$				

[7] Presuming that 'dog' is not also used as a mass-measure, in which case 'three dogs' is ambiguous. Words like 'dollar' have this ambiguous nature; dollars are individuals (as in dollar-bills and dollar-coins), and measures, although the former are becoming extinct in the modern (modem?) economy.

As before, when a CNP [type $D_0 \rightarrow S$] serves as an NP, we convert it into an entity-sum [type ΣD].

4. A Complication

The above account of numerical quantifiers seems nice and tidy, but it is not the whole story. Suppose I ask you how many dogs you own, and you answer as follows.

(A) I own three dogs

I should be entitled to infer that you don't own four dogs. On the other hand, according to the account of 'three' given above, (A) is true precisely if you own at least one dog-trio. But, presuming that dog-ownership is distributive,[8] if you own four dogs, then you also own four dog-trios,[9] and hence at least one dog-trio, so you own three dogs.

The question then is whether my inference is (partly) **pragmatic** or (purely) **semantic**. On pragmatic grounds (conversational protocol), your answer *should* be *maximally informative*, so if you own four dogs, saying you own three dogs is truthful in a "lawyerly" way, but not maximally informative, and is therefore misleading. However, this is a violation of pragmatic standards, not semantic standards.[10]

For this reason, English provides more precise language by which to communicate in a manner that reduces the dependence on pragmatics. In particular, in order to avoid confusion you might say

(A_2) I own **exactly** three dogs,
or (A_3) I own **at least** three dogs.

Whereas 'three' by itself behaves like an adjective, the latter two phrases 'exactly three' and 'at least three' behave like quantifiers. For example, the phrases

the **three** dogs
my **four** dogs
all **five** dogs
no **two** dogs are exactly alike
we **three** kings

become infelicitous when we insert 'exactly' or 'at least', as seen in the following examples.

the **at least three** dogs
my **exactly four** dogs
all **at least five** dogs
no **exactly two** dogs are exactly alike
we **at least three** kings

---

[8] In other words, if one owns a plurality of dogs, then one owns every sub-plurality of those dogs.
[9] Let $\{d_1, d_2, d_3, d_4\}$ be a set of four dogs; then the following are four subsets consisting of three dogs each – $\{d_1, d_2, d_3\}$, $\{d_1, d_2, d_4\}$, $\{d_1, d_3, d_4\}$, $\{d_2, d_3, d_4\}$.
[10] Similarly, if I tell you it is raining or sleeting when in fact I know it is raining but not sleeting, then I am being truthful but misleading.

5. Numerical Quantifiers – Junctions

Whereas ‘one’, ‘two hundred’, etc., are best understood as adjectives rather than quantifiers, ‘exactly one’ and ‘at least two hundred’ are best understood as quantifiers. So, how do we semantically render the phrases ‘exactly’ and ‘at least’?

Earlier, we saw how to analyze various quantifiers – including ‘every’, ‘some’, ‘no’, and ‘any’ – in terms of infinitary-operators (a.k.a. junctions). The same approach can be used for numerical quantifiers. First, the notion of ‘at least’ [as attached to number-words[11]] can be analyzed as follows.

$$[\![\text{at least}]\!] \quad = \quad \lambda \mathbf{N}_0\, \lambda \mathrm{P}_0 \bigvee\{\, x \mid \mathbf{N}(\mathrm{P})[x] \,\}$$

Here the variable ‘**N**’ ranges over numerical-adjectives in the meta-language. The following are example calculations.

1. Jay owns at least three dogs

<table>
<tr><td>Jay [+1]</td><td>owns</td><td>at–least</td><td>three</td><td>dogs</td><td>[+2]</td></tr>
<tr><td></td><td></td><td>$\lambda \mathbf{N}_0\, \lambda \mathrm{P}_0 \bigvee\{\, y \mid \mathbf{N}(\mathrm{P})[y] \,\}$</td><td>$\lambda \mathrm{P}_0 \lambda x_0\, \mathbf{3}(\mathrm{P})[x]$</td><td></td><td></td></tr>
<tr><td></td><td></td><td colspan="2">$\lambda \mathrm{P}_0 \bigvee\{\, y \mid \mathbf{3}\mathrm{P}y \,\}$</td><td>$\mathbf{D}_0$</td><td></td></tr>
<tr><td></td><td></td><td colspan="3">$\bigvee\{\, y \mid \mathbf{3D}y \,\}$</td><td>$\lambda x\{x_2\}$</td></tr>
<tr><td></td><td>$\lambda y_2 \lambda x_1 \mathbf{O}xy$</td><td colspan="4">$\bigvee\{\, y_2 \mid \mathbf{3D}y \,\}$</td></tr>
<tr><td>J_1</td><td colspan="5">$\bigvee\{\, \lambda x_1 \mathbf{O}xy \mid \mathbf{3D}y \,\}$</td></tr>
<tr><td colspan="6">$\bigvee\{\, \mathbf{O}\text{J}y \mid \mathbf{3D}y \,\}$</td></tr>
<tr><td colspan="6">$\exists y\, \{\, \mathbf{3D}y \;\&\; \mathbf{O}\text{J}y \,\}$</td></tr>
</table>

Notice that the output node reads this sentence as semantically-equivalent to ‘Jay owns three dogs’, although the sentence's pragmatic-implications are weaker.

2. at least one dog is barking

<table>
<tr><td>at–least</td><td>one</td><td>dog</td><td>[+1]</td><td>is barking</td></tr>
<tr><td>$\lambda \mathbf{N}_0\, \lambda \mathrm{P}_0 \bigvee\{\, x \mid \mathbf{N}(\mathrm{P})[x] \,\}$</td><td>$\lambda \mathrm{P}_0 \lambda x_0\, \mathbf{1}(\mathrm{P})[x]$</td><td></td><td></td><td></td></tr>
<tr><td colspan="2">$\lambda \mathrm{P}_0 \bigvee\{\, x \mid \mathbf{1}\mathrm{P}x \,\}$</td><td>$\mathbf{D}$</td><td></td><td></td></tr>
<tr><td colspan="3">$\bigvee\{\, x \mid \mathbf{1D}x \,\}$</td><td>$\lambda x\{x_1\}$</td><td></td></tr>
<tr><td colspan="4">$\bigvee\{\, x_1 \mid \mathbf{1D}x \,\}$</td><td>$\lambda x_1 \mathbf{B}x$</td></tr>
<tr><td colspan="5">$\bigvee\{\, \mathbf{B}x \mid \mathbf{1D}x \,\}$</td></tr>
<tr><td colspan="5">$\exists x\{\mathbf{1D}x \;\&\; \mathbf{B}x\}$</td></tr>
</table>

Notice that the output node reads this sentence as semantically-equivalent to ‘some dog is barking’, so long as we presume in the latter that ‘dog’ means dog-individual. The ‘at least one’ locution makes this presumption completely clear.[12]

[11] The expression ‘at least’ is an *all-purpose* adverb; at least, that's what I've heard.
[12] Besides, the prospect of dog-matter barking is very creepy.

Having dealt with 'at least', we next consider 'exactly' [as used with number-words[13]]. In order to deal with this, we introduce yet another infinitary-operator, $\boxtimes$, which corresponds to (anadic) **exclusive-or**, which has the following semantic property.

$\boxtimes\{P_1, \ldots, P_k\}$ is true

iff

exactly one of $\{P_1, \ldots, P_k\}$ is true

We next offer the following analysis of 'exactly'.

$$[\![\text{exactly}]\!] \quad = \quad \lambda\mathbf{N}_0\, \lambda P_0\, \boxtimes\{\, x \mid \mathbf{N}(P)[x] \,\}$$

For example,

(1) $[\![\text{exactly one dog}]\!] \quad = \quad \boxtimes\{\, x \mid \mathbf{1D}x \,\}$

(2) $[\![\text{exactly three dogs}]\!] \quad = \quad \boxtimes\{\, x \mid \mathbf{3D}x \,\}$

etc.

where the right hand sides of (1) and (2) respectively amount to:

$$\text{1-dog}_1 \boxtimes \text{1-dog}_2 \boxtimes \ldots \boxtimes \text{1-dog}_k$$
$$\text{3-dog}_1 \boxtimes \text{3-dog}_2 \boxtimes \ldots \boxtimes \text{3-dog}_k$$

where $\{\text{1-dog}_1, \text{1-dog}_2, \ldots, \text{1-dog}_k\}$ is the set of all *solo*-dogs, $\{\text{3-dog}_1, \text{3-dog}_2, \ldots, \text{3-dog}_k\}$ is the set of all dog-trios, and $\boxtimes$ is the exclusive-or operator.[14]

The following is an example calculation.

3. Jay owns exactly three dogs

Jay [+1]	owns	exactly	three	dogs	[+2]	
		$\lambda\mathbf{N}_0\lambda P_0\boxtimes\{\, y \mid \mathbf{N}(P)[y] \,\}$	$\mathbf{3}_0$			
		$\lambda P_0\boxtimes\{\, y \mid \mathbf{3}Py \,\}$		$\mathbf{D}_0$		
		$\boxtimes\{\, y \mid \mathbf{3D}y \,\}$			$\lambda x\{x_2\}$	
	$\lambda y_2\lambda x_1\mathbf{O}xy$	$\boxtimes\{\, y_2 \mid \mathbf{3D}y \,\}$				
J_1	$\boxtimes\{\, \lambda x_1\mathbf{O}xy \mid \mathbf{3D}y \,\}$					
$\boxtimes\{\, \mathbf{O}Jy \mid \mathbf{3D}y \,\}$						①
$\exists!y\{\, \mathbf{3D}y\ \&\ \mathbf{O}Jy \,\}$						②
$3y\{\mathbf{D}y\ \&\ \mathbf{O}Jy\}$						③

Line ① amounts to the following.

$$\mathbf{O}[J, \text{dog-trio}_1] \boxtimes \ldots \boxtimes \mathbf{O}[J, \text{dog-trio}_k]$$

13 'exactly' is an adverb with wide application – like, but not exactly like, 'at least'.

14 Notwithstanding the formatting of $\boxtimes$, it is **not a binary-operator**, but rather an **anadic-operator** which is formatted so that a "copy" is placed between each argument and the next. It is well-known that reducing this anadic-operator to a dyadic exclusive-or operator produces undesirable (even ridiculous) results.

Given the nature of exclusive-or, this amounts to saying that there is exactly one dog-trio that Jay owns. Now, this is also precisely what ② says, but using the familiar logical notation '∃!' ["there is exactly one…"]. The latter move is underwritten by the following type-logical principle.

$\boxtimes_{v}\{\Psi \mid \Phi\}$ / $\exists! v\{\Phi \,\&\, \Psi\}$	[⊠-simplification]

Here, we have the following first-order definition.

$$\exists!\mathbf{v}\Phi \quad =_{df} \quad \exists\mathbf{w}\forall\mathbf{v}\{\Phi \leftrightarrow \mathbf{v}=\mathbf{w}\}$$

Here, **v** is any variable, and **w** is any variable that is distinct from **v** and not free in Φ.

Finally, line ③ is obtained from ② in accordance with notation and principles explained in the following section.

6. Second-Order Numerical-Predicates

Thus far, we use numerals in the meta-language as predicate-modifiers [type: D→S.→D→S], informally characterized as follows.

1(P)[α] =: α is one P
2(P)[α] =: α are two P
etc.

We can also use numerals in the meta-language as second-order predicates [type: (D→S)→S], characterized as follows.

$\mathbb{1}$[P] =: there is exactly one item that has property P
$=_{df}$ $\exists x\forall y\{Py \leftrightarrow y=x\}$

$\mathbb{2}$[P] =: there are exactly two items that have property P
$=_{df}$ $\exists x_1\exists x_2\{\ x_1\neq x_2 \,\&\, \forall y\{Py \leftrightarrow .\ y=x_1 \vee y=x_2\}\ \}$

$\mathbb{3}$[P] =: there are exactly three items that have property P
$=_{df}$ $\exists x_1\exists x_2 x_3\{\ x_1\neq x_2 \,\&\, x_1\neq x_3 \,\&\, x_2\neq x_3 \,\&$
$\forall y\{Py \leftrightarrow .\ y=x_1 \vee y=x_2 \vee y=x_3\}\ \}$

etc.

We also introduce the following natural shorthand,

$$\mathbb{N}\nu\Phi \quad =_{df} \quad \mathbb{N}[\lambda\nu\Phi]$$

where ν is a variable, Φ is a formula, and $\mathbb{N}$ is a second-order numerical-predicate (as above).

Note carefully that these definitions only make sense when talking about count-objects. Of course, from the viewpoint of the meta-language, all items are count-items, including those items we use to *model* mass-items. For example, we have the following translation, in effect.

α is land ⌒ α is **a** mass-entity consisting of land

This means that, although we cannot count lands in the object-language, we can count land-entities in the meta-language, and of course there are uncountably-many of the latter, supposing land behaves like geometrical surfaces. So, if you own land, you own uncountably-many land-entities, but of course this does not mean that you own an infinite amount of land.

7. Scope Ambiguity

When quantifier-phrases are combined, we often get scope-ambiguities, and numerical-quantifiers are no exception, as illustrated in the following example.

exactly-two men respect exactly-two women

Indeed, this has **three** readings, since it can be used to answer three quite different questions.[15]

1. how many men respect exactly-two women?
2. how many women are respected by exactly-two men?
3. how many men respect how many women?

The first two are computed respectively as follows.

4. exactly two men respect exactly two women ['exactly two men' has wide scope]

<table>
<tr><td>exactly two men [+1]</td><td>respect</td><td>exactly two woman [+2]</td><td></td></tr>
<tr><td rowspan="2">⊠{ x_1 | $\mathbf{2M}x$ }</td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>⊠{ y_2 | $\mathbf{2W}y$ }</td><td></td></tr>
<tr><td colspan="2">⊠{ $\lambda x_1 \mathbf{R}xy$ | $\mathbf{W}y$ & $\mathbf{2}y$ }</td><td>①</td></tr>
<tr><td colspan="3">⊠{ ⊠{ $\mathbf{R}xy$ | $\mathbf{2W}y$ } | $\mathbf{2M}x$ }</td><td>②</td></tr>
<tr><td colspan="3">∃!x { $\mathbf{2M}x$ & ∃!y{$\mathbf{2W}y$ & $\mathbf{R}xy$}}</td><td></td></tr>
<tr><td colspan="3">$2x${ $\mathbf{M}x$ & $2y${$\mathbf{W}y$ & $\mathbf{R}xy$} }</td><td></td></tr>
</table>

Here, in deriving ② from ①, the first junction absorbs the second junction. By contrast, in the following, the second junction absorbs the first junction.

5. exactly two men respect exactly two women ['exactly two women' has wide scope]

<table>
<tr><td>exactly two men [+1]</td><td>respect</td><td>exactly two woman [+2]</td><td></td></tr>
<tr><td rowspan="2">⊠{ x_1 | $\mathbf{M}x$ & $\mathbf{2}x$ }</td><td>$\lambda y_2 \lambda x_1 \mathbf{R}xy$</td><td>⊠{ y_2 | $\mathbf{2W}y$ }</td><td></td></tr>
<tr><td colspan="2">⊠{ $\lambda x_1 \mathbf{R}xy$ | $\mathbf{2W}y$ }</td><td>①</td></tr>
<tr><td colspan="3">⊠{ ⊠{ $\mathbf{R}xy$ | $\mathbf{2M}x$ } | $\mathbf{2W}y$ }</td><td>②</td></tr>
<tr><td colspan="3">∃!y{ $\mathbf{2W}y$ & ∃!x{$\mathbf{2M}x$ & $\mathbf{R}xy$} }</td><td></td></tr>
<tr><td colspan="3">$2y${ $\mathbf{W}y$ & $2x${$\mathbf{M}x$ & $\mathbf{R}xy$} }</td><td></td></tr>
</table>

The third reading is considerably more complicated and corresponds to a non-linear reading of the quantifier-phrases, according to which neither gains wide-scope over the other; rather, the quantifier-phrases act in parallel. Coming up with a logical formula that conveys this reading is not overly difficult. Here it is.

3. $2x\{\mathbf{M}x \,\&\, \exists y\{\mathbf{W}y \,\&\, \mathbf{R}xy\}\} \,\&\, 2y\{\mathbf{W}y \,\&\, \exists x\{\mathbf{M}x \,\&\, \mathbf{R}xy\}\}$

What is difficult, as usual, is to accomplish this compositionally – *deriving* the proposed semantic-value from the semantic-values of the morphemes.

[15] We presume a simple-minded account of *respect* according to which it stands between individuals and not groups of individuals. Otherwise, there are further (collective) readings as well. Also, 'respect' is stative. When eventive verbs and nouns are involved, counting is even more complicated. For example, what does the following mean? Last year, the world's busiest airport was Atlanta's airport (ATL), which served 90,039,280 passengers, which was followed by Chicago's airport (ORD), which served 69,353,654 passengers.