Trees

Trees (Blackburn et al. 2001: p. 6)

Definition 1.7 A *tree* $\mathfrak{T}$ is a relational structure (T, S) where:

(i) T, the set of nodes, contains a unique $r \in T$ (called the *root*) such that $\forall t \in T S^* rt$.

(ii) Every element of T distinct from r has a unique S-predecessor; that is, for every $t \neq r$ there is a unique element $t' \in T$ such that $St't$.

(iii) S is *acyclic*; that is, $\forall t \neg S^+ tt$. (It follows that S is irreflexive.)

Finite transitive trees: (Blackburn et al. 2001: p. 6)

A *transitive tree* is an SPO $(T, <)$ such that (i) there is a *root* $r \in T$ satisfying $r < t$ for all $t \in T$ and (ii) for each $t \in T$, the set $\{s \in T \mid s < t\}$ of predecessors of t is finite and linearly ordered by $<$.

Finite binary branching trees (Blackburn and Meyer-Viol 1997: p. 30)

A tuple $\langle W, \succ_1, \succ_2, \mathit{root}, \Theta \rangle$ where

> "W is a finite, non-empty set, the set of tree nodes;
>
> $\Theta (\subseteq W)$ contains all and only the tree's terminal nodes;
>
> and *root* is the (unique) root node of the tree.
>
> As for $\succ_1$ and $\succ_2$, there are binary relations defined as follows:
>
> (a) for all $w, w' \in W$, $w \succ_1 w'$ iff w' is the first daughter of w; and
>
> (b) $w \succ_2 w'$ iff w' is the second daughter of w.
>
> (c) Note that $\succ_1$ and $\succ_2$ are partial functions, for any node in an ordered binary tree has at most one first daughter and a most one second daughter.
>
> (d) Further note that if $w \succ_2 w'$ then there exists a unique w'' such that $w \succ_1 w''$.
>
> (e) Moreover, $w'' \neq w$."

Trees: (Carpenter 1997: p. 520-523)

"We construct trees from a set **BasExp** of basic expressions as well as a set **Cat** of *categories*, which are used to classify expressions. [...] The set **Tree** of *trees* over the basic set **BasExp** of basic expressions and the set **Cat** of *categories* is the least such satisfying the following:

(40) a. $\mathbf{BasExp} \subseteq \mathbf{Tree}$

b. $\langle C, \langle T_1, \ldots, T_n \rangle \rangle \in \mathbf{Tree}$ if $n \geq 0, C \in \mathbf{Cat}, T_1, \ldots, T_n \in \mathbf{Tree}$

Note that as long as **BasExp** and **Cat** are nonempty, **Tree** is an infinite set each of whose members is a finite structure. Furthermore, if both sets are countable, then so is **Tree**."

Tree (McCawley 1968/1976: p. 36-37)

"Before giving the procedure for constructing a tree from a derivation, it will be necessary to give a definition of the notion 'tree'. A tree is a finite set of obects (called 'nodes') with three relationships ρ 'directly dominates', λ 'is to the left of', and α 'bears the label', satisfying the following axioms:

(1) there is a node x_0 such that for no node x does $x \rho x_0$ (x_0 is called the 'root' of the tree);

(2) if x is a node distinct from x_0 then $x_0 \rho^* x$, where ρ^* is the relationship which holds between two nodes a and b if there is a chain of nodes $a_1, \ldots, a_n$ such that $a \rho a_1, a_1 \rho a_2, \ldots a_n \rho b$ (ρ^* can be read 'dominates'; this axiom asserts that a tree is 'connected');

(3) if $x \rho y$ and $x' \rho y$, then $x = x'$ (i.e. a tree contains no 'loops';

(4) λ is a partial ordering on the nodes (i.e. if $x \lambda y$ and $y \lambda z$, then $x \lambda z$; if $x \lambda y$, then it is false that $y \lambda x$);

(5) for any two nodes x and y, if $x \neq y$, then either $x \rho^* y$ or $y \rho^* x$ or $x \lambda y$ or $y \lambda x$;

(6) if x is a non-terminal (i.e. if there is a z such that $x \rho z$) and $x \lambda y$, then there is an x' such that $x \rho x'$ and $x' \lambda y$;

(7) every node bears the relation α to exactly one element (its 'label'), the possible labels being a set of objects distinct from the nodes."

0.0.1 Trees (?: p. 50)

"... a *tree* is a nonempty prefix-closed subset of ω^*. In other words, it is a set T of finite-length strings of natural numbers such that

- $\epsilon \in T$;
- if $xy \in T$, then $x \in T$.

A *path* in T is a maximal subset of T linearly ordered by the prefix relation. The tree T is *well-founded* if it has no infinite paths; equivalently, if the converse of the prefix relation is a well-founded relation on T."

References

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