III. Simple Regression.

A. Introduction
B. Population Regression Equation
C. Sample Regression Equation
D. Ordinary Least Squares
E. Classical Regression Model.
F. Properties of OLS Estimators (**BLUE**)
G. Estimator for σ^2
H. Inference (we've done this before!)

F. Properties of OLS Estimators

- What properties do we want for our estimators? ***Unbiased and Min. Variance***
- What are we trying to describe? ***Sampling Distribution***
- Use expected values to determine:

  Center – $E[\hat{\beta}_1] = \beta_1$

  Variance – $Var(\hat{\beta}_1) = E[(\hat{\beta}_1 - \beta_1)^2]$

OLS Estimators

$$\hat{\beta}_1 = \frac{\sum X_i Y_i - n\bar{X}\bar{Y}}{\sum X_i^2 - n\bar{X}^2} = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{\sum x_i Y_i}{\sum x_i^2}$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

Given the CRM Assumptions, the ***Sampling Distributions*** have the following ***centers*** and ***variances***:

$$E[\hat{\beta}_0] = \beta_0 \qquad \sigma_{\hat{\beta}_0} = \sqrt{\frac{\sigma^2 \sum X_i^2}{n \sum x_i^2}}$$

$$E[\hat{\beta}_1] = \beta_1 \qquad \sigma_{\hat{\beta}_1} = \sqrt{\frac{\sigma^2}{\sum x_i^2}}$$

CRMA # 6: We assume that the disturbances are normally distributed.

- *Linear combinations of normally distributed variables are also normally distributed.*
- Y is a linear combination of u.
- OLS estimators - linear combinations of Y.
- OLS estimators are normally distributed.

G. Estimator for σ^2

- We will never know σ^2 in the real world.
- σ^2 is an unknown population parameter.
- What would be a logical estimator?
- Recall:

H. Inference in Simple Regression

1. **Introduction:** What we have, what we know.
   - ***PRE:***
   - ***Point Estimation***:
   - ***Inference***:

H. Inference (cont.)

2. σ^2 known

- Calculate the *true standard error* for the sampling distribution.
- ***CI formula***: what is it?
- Hypothesis test: Z_{calc} vs. $Z_{\alpha/2}$ or Z_{α}

PRS : On today's handout, we've listed all of our sampling experiment results, estimates and confidence intervals. Do your confidence intervals contain the true value for β_1? (β_1 = \$ 0.70)

1. Yes, both my intervals contain the true value.
2. One of my intervals contains the true value, one does not.
3. Neither of my intervals contain the true value.

H. Inference (cont.)

3. σ^2 NOT known

- Estimate the *true standard error*
- ***CI formula***: what is it?
- Standardized test: t_{calc} vs. $t_{\alpha/2}$ or t_{α}

4. Hypothesis tests: Six Steps

Step 1: Hypotheses. Start with the ***alternative*** or ***research hypothesis***.

Tests for β_1 – what does theory suggest for the effect of X on Y? This determines your alternative hypothesis.

Examples:

X is commodity price, Y is quantity demanded.

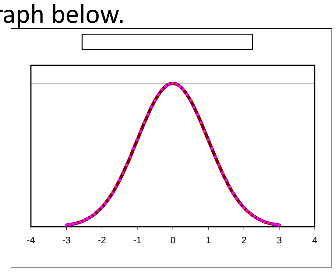
X is commodity price, Y is quantity supplied.

X price of a substitute, Y is quantity demanded.

Step 2: Choose α – ***level of significance***.

Step 3: Determine ***critical value(s).*** Draw!

- What kind of test? Label critical regions on the graph below.



Step 4: Calculate the test statistic – a t_{calc} or z_{calc}.

For Example:

$$t_{calc} = \frac{\hat{\beta}_1 - \beta_1}{s_{\hat{\beta}_1}}$$

Step 5: Compare – where does your test statistic fall? Which region?

Step 6: Conclusion.

Fail to Reject: Your estimate ***is not statistically different from the hypothesized value.***

Reject: Your estimate ***is statistically different from the hypothesized value.***