IV. Multiple Regression.

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Elasticities – another interpretation of results.

- Elasticity – the % change in quantity due to a 1% change in price, or income.
- From micro:

$$\varepsilon_p = \left(\frac{\Delta Sales}{\Delta Prose}\right) \cdot \frac{Prose}{Sales}$$

- For a regression model:

$$\varepsilon_p = \left(\frac{\partial Sales}{\partial Prose}\right) \cdot \frac{Prose}{Sales} = \beta_1 \cdot \frac{Prose}{Sales}$$

Elasticities – another interpretation of results.

- What values do we use for ***Prose*** and ***Sales***?
- Observed values:

- Sample Means:

E. Properties of OLS Estimators

1. ***Linear:*** The OLS estimators are still linear estimators.
2. ***Unbiased:*** If CRMAs # 1 – 3 are correct, the OLS estimators are unbiased estimators.
3. ***Minimum Variance:*** If CRMAs # 1 – 5 are correct, the OLS estimators are the ***Best Linear Unbiased Estimators.***

   (Gauss-Markov Theorem)

   Draw a graph illustrating properties 2 & 3.

F. Estimator for σ^2 and Variances of $\hat{\beta}s$

1. Our ***simple regression*** estimator for σ^2:

$$\hat{\sigma}^2 = \frac{\sum e_i^2}{n-2}$$

2. What changes in ***multiple regression***?
   - Still use e_i to calculate estimate of σ^2
   - Degrees of freedom change – we estimate ***K*** "slope parameters."

$$\hat{\sigma}^2 = \frac{\sum e_i^2}{n-K-1}$$

F. Estimator for σ^2 and Variances of $\hat{\beta}s$

1. Our ***simple regression*** estimator for σ^2:
2. What changes in ***multiple regression***?

3. Standard errors for $\hat{\beta}s$
   - Formula – i.e.,

$$s_{\hat{\beta}_1} = \sqrt{\frac{\hat{\sigma}^2 \sum x_{2i}^2}{\left(\sum x_{1i}^2\right)\left(\sum x_{2i}^2\right) - \left(\sum x_{1i}x_{1i}\right)^2}}$$

$$s_{\hat{\beta}_2} = \sqrt{\frac{\hat{\sigma}^2 \sum x_{1i}^2}{\left(\sum x_{1i}^2\right)\left(\sum x_{2i}^2\right) - \left(\sum x_{1i}x_{1i}\right)^2}}$$

Yuck! But the software takes care of these.

G. Inference in Multiple Regression.

1. Confidence Intervals – same deal:

2. Hypothesis Tests – Standardized Tests.

Regression Analysis: sales versus prose, pcarn, dinc

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The regression equation is
sales = 13355 – 3628 prose + 2634 pcarn - 19.3 dinc

Predictor     Coef  SE Coef      T      P
Constant     13355     6485   2.06  0.062
prose      -3628.2    635.6  -5.71  0.000
pcarn         2634     1013   2.60  0.023
dinc        -19.25    30.69  -0.63  0.542

S = 1076.29  R-Sq = 77.8%  R-Sq(adj) = 72.2%
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H. Goodness of Fit – R^2

1. Multiple Coefficient of Determination – R^2
2. Calculation – exactly the same.

   R^2 = ESS / TSS = 1 – (RSS / TSS)
3. Adjusted R^2:

$$\bar{R}^2 = 1 - \frac{RSS}{TSS}\left(\frac{n-1}{n-K-1}\right)$$

R^2 – used for assessing how well ***a model*** fits.
Adj. R^2 – used to ***compare two models***.
Same sample – same Y values.