V. Extensions of Multiple Regression

A. Dummy Variables

B. Scaling Variables

C. Non-Linear Models

1. **Basic Idea for Estimation**.
2. **Models that don't require logs**
3. **Models that require log transform**.

C. Non-Linear Models

1. ***Basic idea***: Make the model look linear in the parameters. Then estimate using OLS.
2. ***Models that don't require a log transform***.

Quadratic form:

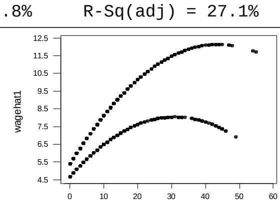
Examples:

Interpretation:

```
wage = - 5.49 + 0.905 ed + 0.309 ex - 0.00352 exsq
      - 0.726 fe - 0.0903 feex
```

Predictor	Coef	StDev	T	P
Constant	-5.494	1.2140	-4.53	0.000
ed	0.9046	0.0791	11.43	0.000
ex	0.3093	0.0554	5.59	0.000
exsq	-0.0035	0.0012	-2.98	0.003
fe	-0.7264	0.6725	-1.08	0.281
feex	-0.0903	0.0309	-2.93	0.004

S = 4.388   R-Sq = 27.8%   R-Sq(adj) = 27.1%



C. Non-Linear Models

Inverse form:

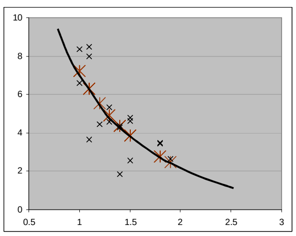
Example:

Interpretation:

```
The regression equation is
w-dot = - 1.04 + 7.91 INV(U)

Predictor       Coef     SE Coef          T        P
Constant      -1.044       2.373      -0.44    0.666
INV(U)         7.907       3.164       2.50    0.025

S = 1.734       R-Sq = 29.4%     R-Sq(adj) = 24.7%
```



C. Non-Linear Models

3. Models that require a log transform.

- ***Cobb-Douglas***: log-log models.

$$Y = AX_1^{\beta_1} X_2^{\beta_2} \exp^u$$

- Transform using natural logarithm.
- Interpretation:

***3. Models that require a log transform*.**

- ***Semi-Log models***: log-linear

$$Y = \exp^{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + u}$$

- Transform using natural logarithm.
- Parameters:

***3. Models that require a log transform*.**

- ***Semi-Log models***: linear-log.

$$\exp^{Y} = A X_1^{\beta_1} X_2^{\beta_2} \exp^{u}$$

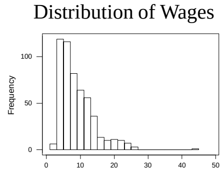
- Transform using natural logarithm.
- Parameters:

4. Other Items for nonlinear models:

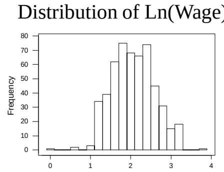
a. CRM Normality Assumption

- What did we assume was normally distributed?
- What part of our analysis relies on this assumption?
- Some data are not normally distributed – wages.
- Log transformation – the natural log is normal.

Distribution of Wages



Distribution of Ln(Wage)



b. Dummy Variables in Log Models.

- Dependent variable is now lnY. Not in the natural units.
- Dummy variable coefficient measures:

  $\delta = \ln Y(D=1) - \ln Y(D=0)$
- Proper interpretation:

  % change in Y = $(\exp^{\delta} - 1)\cdot 100$

```
lnwage = - 0.921 + 0.988 lned + 0.235 lnex + 0.047 fe
        - 0.124 lnexfe

Predictor        Coef       StDev          T        P
Constant      -0.9212      0.2524      -3.65    0.000
lned          0.98774     0.08731      11.31    0.000
lnex          0.23505     0.03111       7.56    0.000
fe             0.0475      0.1256       0.38    0.706
lnexfe       -0.12383     0.04555      -2.72    0.007

S = 0.4471      R-Sq = 27.4%     R-Sq(adj) = 26.8%
```