Announcements

- **Exam 1 – Thursday, Feb. 26**.
  - Exams are in Lab, applied questions and theory.
  - During your regularly scheduled Thursday lab.
  - Friday lab – schedule a time for Thursday evening using the Doodle link on the website.
- Term Project: Select your teammate (1) and a topic by **Today, Feb 23**.

I. Introduction

II. Statistical and Notational Preliminaries

A. Introduction

B. Elements of Statistical Theory

1. Prerequisites: Summarizing distributions, point estimation, interval estimation, hypothesis testing.
2. Bivariate measures: covariance and correlation.
3. Expected Values.
4. Estimators and Desirable Properties

3. Expected Values.

a. Definition - The weighted sum of ***all possible values*** of a random variable, where weights are probabilities or relative frequencies.

b. Measure of ***central tendency –*** average **all possible values** for a random variable – ***Population mean***.

Application of Expected Values: Portfolios

- Assume two investments: X and Y.
- ***Joint distribution*** of returns to $1,000 invested.
- Probabilities – P(*XY*) represent different "states of the economy."

X	*Y*	P(*XY*)
-100	50	0.2
50	30	0.4
200	20	0.3
300	20	0.1

PRS 1: Calculate E[X]. Round to a whole dollar.

$$E(X) = \sum_{i=1}^{N} X_i \, P(X_i Y_i)$$

X	Y	P(XY)	P(XY)·X	P(XY)·Y
-100	50	0.2		
50	30	0.4		
200	20	0.3		
300	20	0.1		

c. Rules for using Expected Values

1. E[*c*] = *c*, where *c* is a constant.
2. E[*cY*] = *c* E[*Y*], where *Y* is a random variable.
3. E[*cY* + *bZ*] = *c* E[*Y*] + *b* E[*Z*] .
4. E[*YZ*] ≠ E[*Y*] E[*Z*] and E[*Y/Z*] ≠ E[*Y*] / E[*Z*]
5. If Y and Z are ***independent***, then:

   E[*YZ*] = E[*Y*] E[*Z*]

d. Using the Rules – Examples:

- The sample mean is a random variable.
- Evaluate the sample mean – average ***all possible sample means*** given sample size n.

- Portfolio Expected Returns:

PRS 2: Given the Expected Returns for X and Y, what weight would you choose for X in your portfolio? Why?

1. 0.10
2. 0.30
3. 0.50
4. 0.70
5. 0.90

e. Other "Famous" Expected values

- ***Population Variance*** – measure of dispersion.

- ***Covariance*** – measure of linear association.

- Risk: Use the formula for expected value to calculate variances:

$$\sigma_Y^2 = \sum_{i=1}^{N} (Y_i - \mu_Y)^2 P(X_i Y_i)$$

$$\sigma_X^2 = \sum_{i=1}^{N} (X_i - \mu_X)^2 P(X_i Y_i)$$

- Are X and Y related? Calculate covariance:

$$\sigma_{XY} = \sum_{i=1}^{N} (Y_i - \mu_Y)(X_i - \mu_X) P(X_i Y_i)$$

- Expected values or expected returns for X and Y:

  E[X] = μ_X =

  E[Y] = μ_Y = $30.

- Standard deviations and covariance:

  σ_X = $126.10

  σ_Y = $ 10.95

  σ_{XY} = $ -1300

- ***Expected returns for a portfolio*** of X and Y, a weighted average of two expected returns:

Choose weights for the two investments.
Portion of total investment in each.

PRS 3*:* Given the **Expected Returns *and* Risks** for X and Y, what weight would you choose for investment X in your portfolio?

1. 0.10
2. 0.30
3. 0.50
4. 0.70
5. 0.90

PRS 4: Given ***your weights***, calculate E[P]. **Round to a whole dollar**.

$$E[P] = w_X \, E(X) + (1 - w_X) E(Y)$$

- ***Portfolio Risk*** – a portfolio of X and Y.

$$\sigma_P = \sqrt{w_X{}^2 \sigma_X^2 + (1 - w_X)^2 \sigma_Y^2 + 2 w_X (1 - w_X) \sigma_{XY}}$$

What are the parts? What's in this formula?